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An Overview of Evaluation Methods in Games

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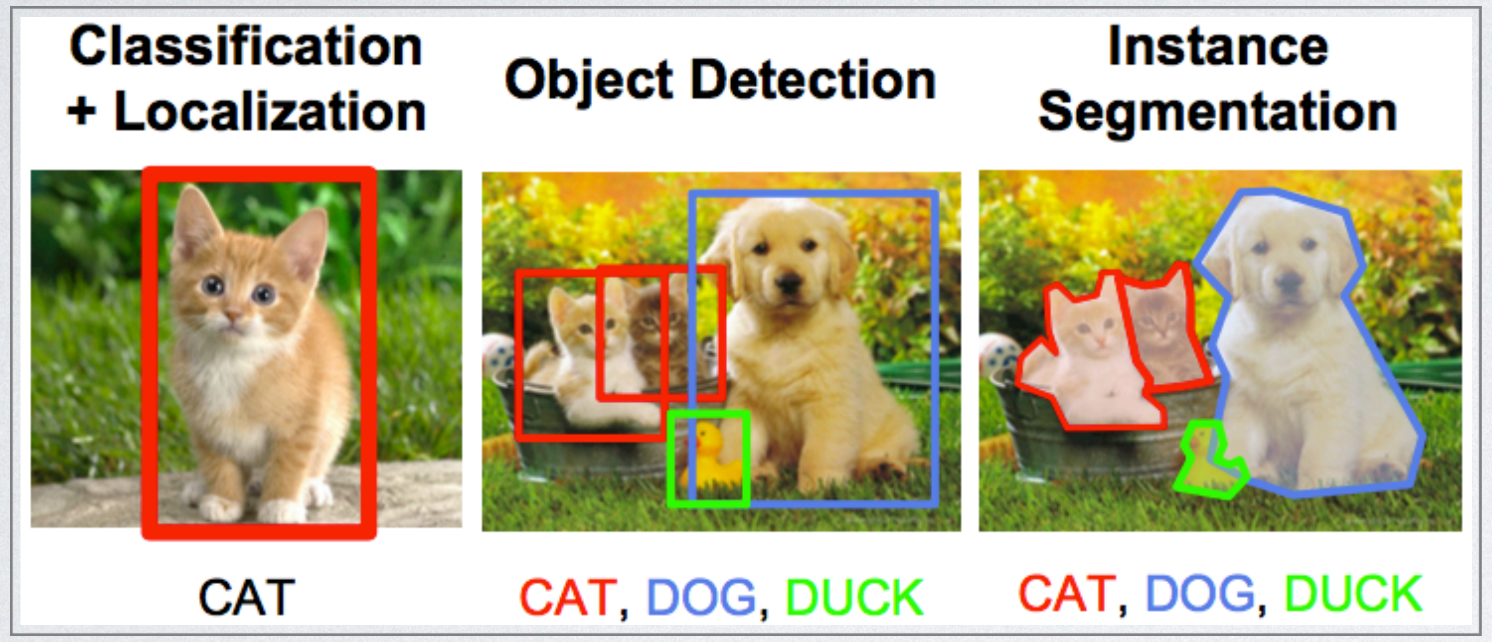
Full Talk address: https://rlchina.org

Why Multi-agent Learning ?

- Reinforcement learning turns data/knowledge into closed-loop decision making.
- Multi-agent learning deal with interactions among the learning agents.



Data

Knowledge

Data

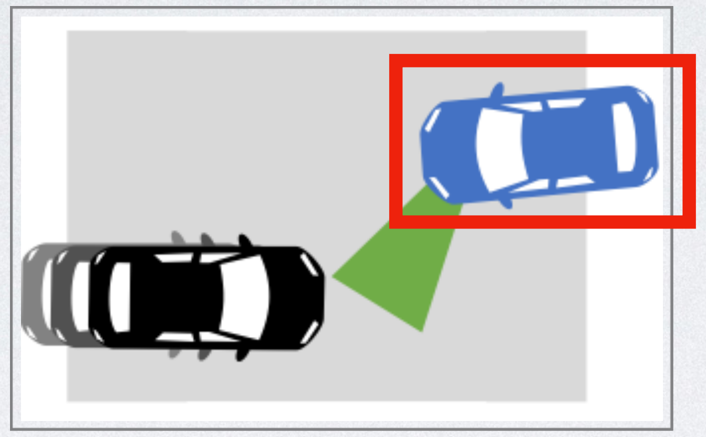
Knowledge

interactions happen!

Decisions

Multi-agent Learning for Autonomous Driving

Traffic intersection is naturally a multi-agent system. From each driver's perspective, in order to perform the optimal action, he must take into account others' behaviours.



scenario

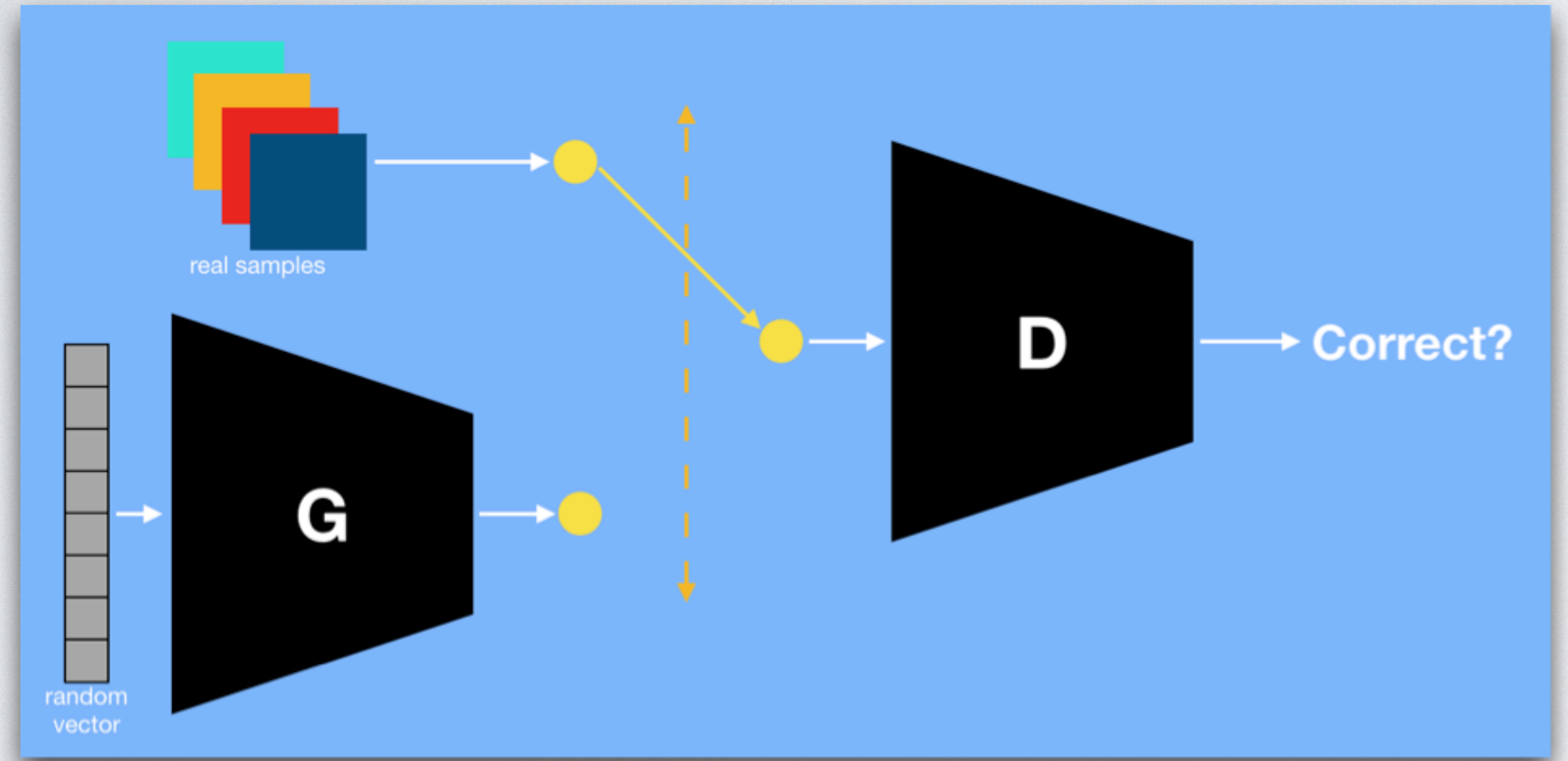
	Yield	Rush
Yield	(0, 0)	(1, 2)
Rush	(2, 1)	(0, 0)

normal-form game

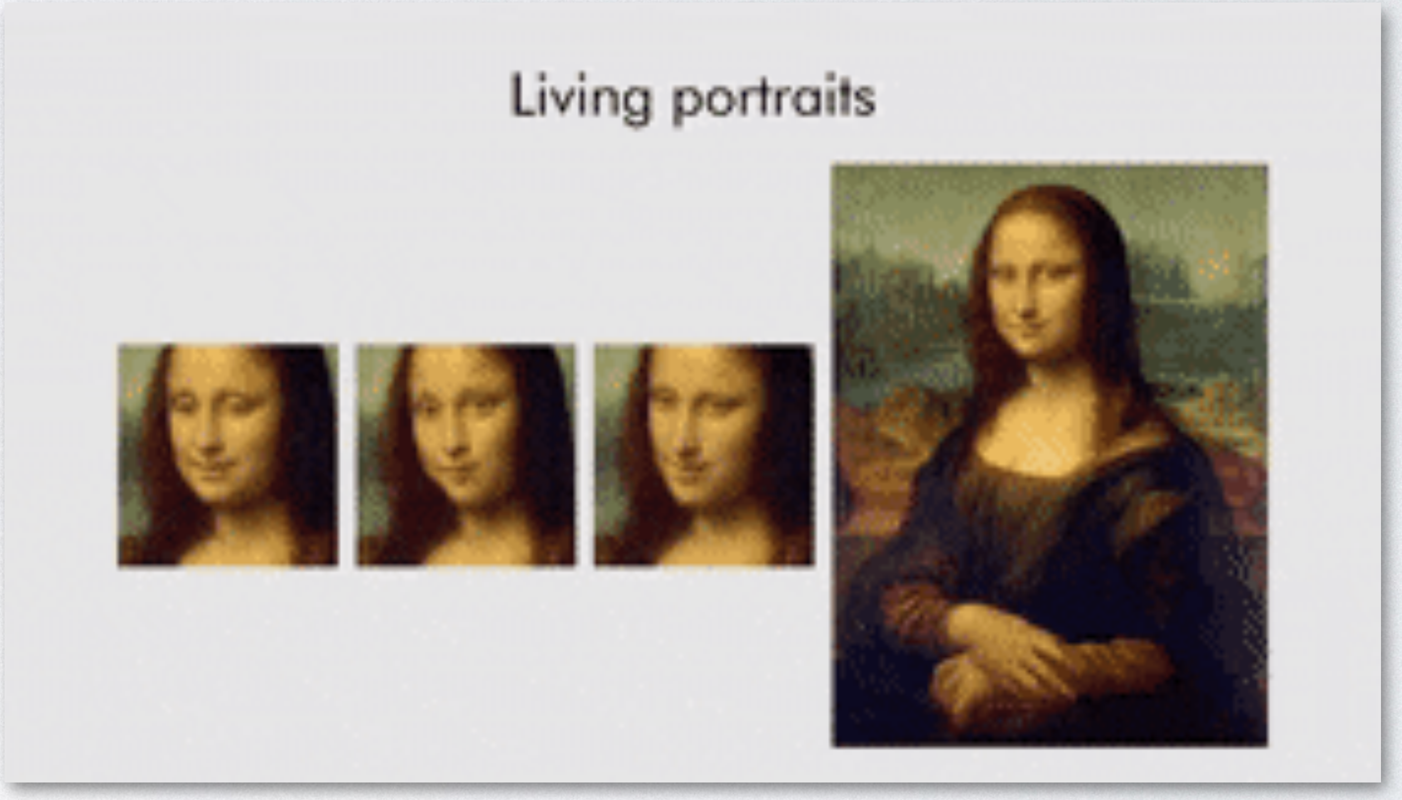
- When the drivers are rational, they will reach the outcome of a Nash Equilibrium. It is the outcome of interaction. Knowing it can predict future.
- Real-world decision making has cooperation & competition. For each agent, how to infer the belief of the other agents and make the optimal action is critical.
- The concept of using traffic light is in fact a correlated equilibrium.
- Many-agent system is when # of agents >> 2. It is a very challenging problem.

Multi-agent Learning for Machine Learning

Two-player zero-sum game → Generative Adversarial Network



CycleGANs



StyleGAN

player 1 | player 2

$$\min_{\theta_g} \max_{\theta_d} \left[\mathbf{E}_{x \sim p_{\text{data}}} \log D_{\theta_d}(x) + \mathbf{E}_{z \sim p(z)} \log\left(1 - D_{\theta_d}\left(G_{\theta_g}(z)\right)\right) \right]$$

Problem Formulation: Singe-agent Reinforcement Learning

- Learn the optimal behaviour through trial-and-errors from the environment.
- Modelled by a Markov Decision Process (MDP) $(\mathcal{S}, \mathcal{A}, \mathcal{R}, \mathcal{T}, \mathcal{P}_0, \gamma)$
  - $\mathcal{S}$ denotes the state space,
  - $\mathcal{A}$ is the action space,
  - $\mathcal{R} = \mathcal{R}(s, a)$ is the reward function,
  - $\mathcal{T} : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \rightarrow [0,1]$ is the state transition function,
  - $\mathcal{P}_0$ is the distribution of the initial state, γ is a discount factor.
- The goal is to find the optimal policy π that maximises expected reward:
  - Discounted reward:

$$V_\pi(s) = \sum_{t=0}^{\infty} \gamma^t \mathbf{E}_{\pi,\mathcal{P}} \left\{ R_t \,|\, s_0 = s, \pi \right\}$$

  - Time-average reward:

$$V_\pi(s) = \lim_{T \rightarrow \infty} \sum_{t=0}^{T} \frac{1}{T} \mathbf{E}_{\pi,\mathcal{P}} \left\{ R_t \,|\, s_0 = s, \pi \right\}$$

Solution to Single-Agent RL

- Value-based method (learn the Q-function $Q(s,a) = r^j(s,a) + \gamma \mathbf{E}_{s' \sim p}[v_\pi(s')]$):

$$Q^{\text{new}}(s_t, a_t) \leftarrow \underbrace{Q(s_t, a_t)}_{\text{old value}} + \underbrace{\alpha}_{\text{learning rate}} \cdot \overbrace{\left(\underbrace{\underbrace{R_t}_{\text{reward}} + \underbrace{\gamma}_{\text{discount factor}} \cdot \underbrace{\max_a Q(s_{t+1}, a)}_{\text{estimate of optimal value}}}_{\text{new value (temporal difference target)}} - \underbrace{Q(s_t, a_t)}_{\text{old value}} \right)}^{\text{temporal difference}}$$

$$\mathcal{H} Q(s,a) = \mathbf{E}_{s'} \left(R(s,a) + \gamma \max_b Q\left(s', b\right) \right)$$ is a contraction-mapping operator.

- Policy-based method (learn the policy $\pi_\theta(\cdot \mid s_t)$ parameterised by θ):

$$J(\theta) = \sum_{s \in S} d^\pi(s) V^\pi(s) = \sum_{s \in S} d^\pi(s) \sum_{a \in \mathcal{A}} \pi_\theta(a \mid s) Q^\pi(s,a), \quad d^\pi(s) = \lim_{t \to \infty} \mathcal{P}\left(s_t = s \mid s_0, \pi_\theta\right)$$

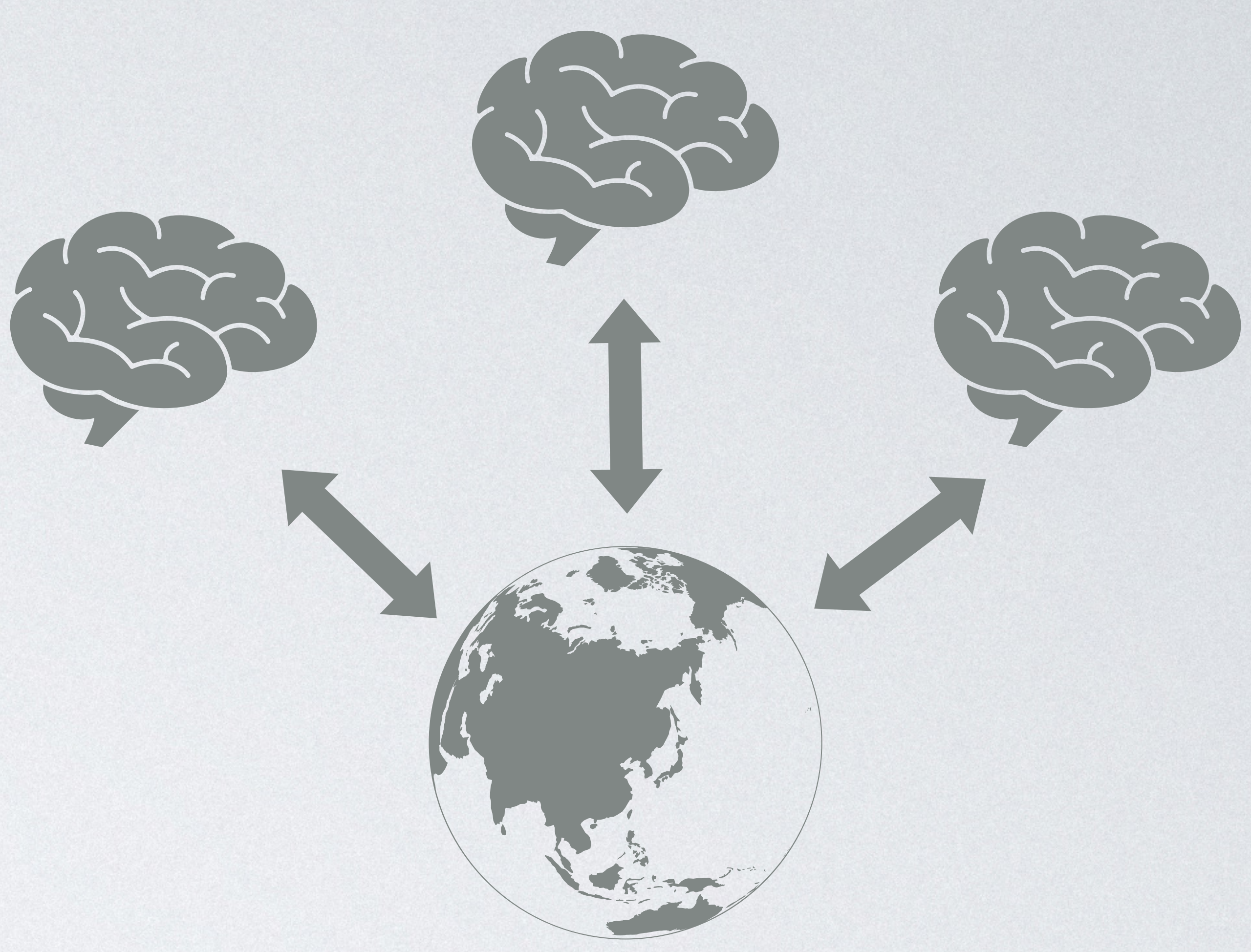
Occupancy measure on state induced by following π_θ in the MDP

$$\Delta\theta \propto \nabla_\theta J(\pi) = \mathbf{E}_{s,a}\left[\nabla_\theta \log \pi(s,a) \cdot Q^\pi(s,a)\right]$$

Push the parameters towards the direction where the reward is large

Problem Formulation: Multi-agent Reinforcement Learning

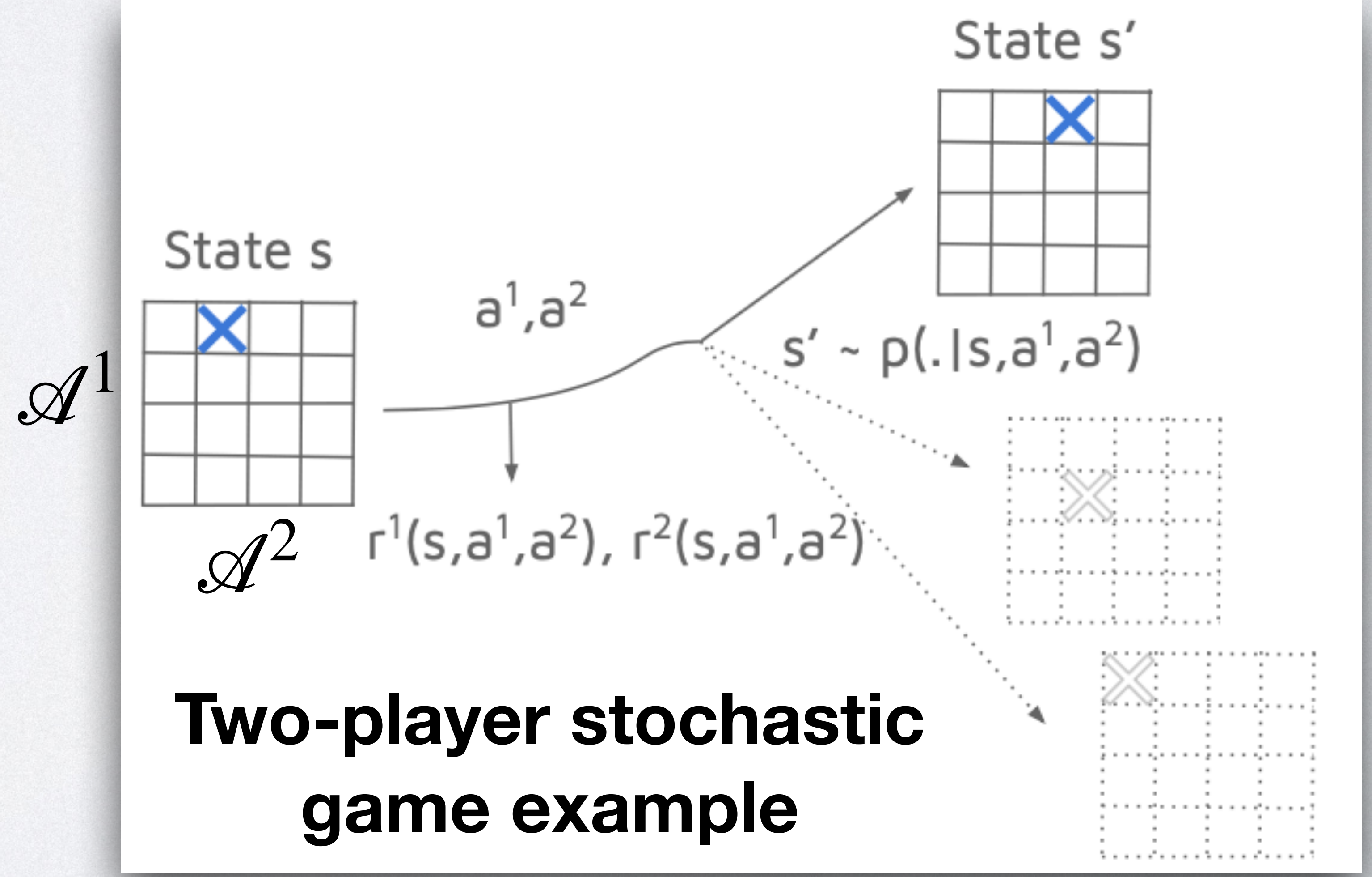
- Modelled by a Stochastic Game $(\mathcal{S}, \mathcal{A}^{\{1,\ldots,n\}}, \mathcal{R}^{\{1,\ldots,n\}}, \mathcal{T}, \mathcal{P}_0, \gamma)$
  - $\mathcal{S}$ denotes the state space,
  - $\boldsymbol{\mathcal{A}}$ is the joint-action space $\mathcal{A}^1 \times \ldots \times \mathcal{A}^n$,
  - $\mathcal{R}^i = \mathcal{R}^i(s, a^i, a^{-i})$ is the reward function for the i-th agent,
  - $\mathcal{T} : \mathcal{S} \times \boldsymbol{\mathcal{A}} \times \mathcal{S} \rightarrow [0,1]$ is the transition function based on the joint action,
  - $\mathcal{P}_0$ is the distribution of the initial state, γ is a discount factor.
  - **Special case:** $n = 1 \rightarrow$ single-agent MDP, $|\mathcal{S}| = 1 \rightarrow$ normal-form game
  - **Dec-POMDP**: assume state is not directly observed, but agents have same reward function.



- Each agent tries to maximise its expected long-term reward:

$$V_{i,\boldsymbol{\pi}}(s) = \sum_{t=0}^{\infty} \gamma^t \mathbf{E}_{\boldsymbol{\pi},\mathcal{P}} \{R_{i,t} \,|\, s_0 = s, \boldsymbol{\pi}\}, \boldsymbol{\pi} = [\pi_1, \ldots, \pi_N]$$

$$Q_{i,\boldsymbol{\pi}}(s, \boldsymbol{a}) = R_i(s, \boldsymbol{a}) + \gamma \mathbf{E}_{s' \sim p} \left[V_{i,\boldsymbol{\pi}}(s')\right]$$



Two-player stochastic game example

Solution to Multi-Agent RL

- Value-based method:
  - The sense of optimality changes, now it depends on other agents !

$$Q_{i,t+1}\left(s_k, \boldsymbol{\pi}_t\right) = Q_{i,t}\left(s_t, \boldsymbol{\pi}_t\right) + \alpha\left[R_{i,t+1} + \gamma \cdot \mathbf{eval}_i\left\{Q_{\cdot,t}(s_{t+1}, \cdot\,)\right\} - Q_{i,t}\left(s_t, \boldsymbol{\pi}_t\right)\right]$$

$$\pi_{i,t}(s, \cdot\,) = \mathbf{solve}_i\left\{Q_{\cdot,t}(s_t, \cdot\,)\right\}$$

    - **Fully-cooperative game**: agents share the same reward function

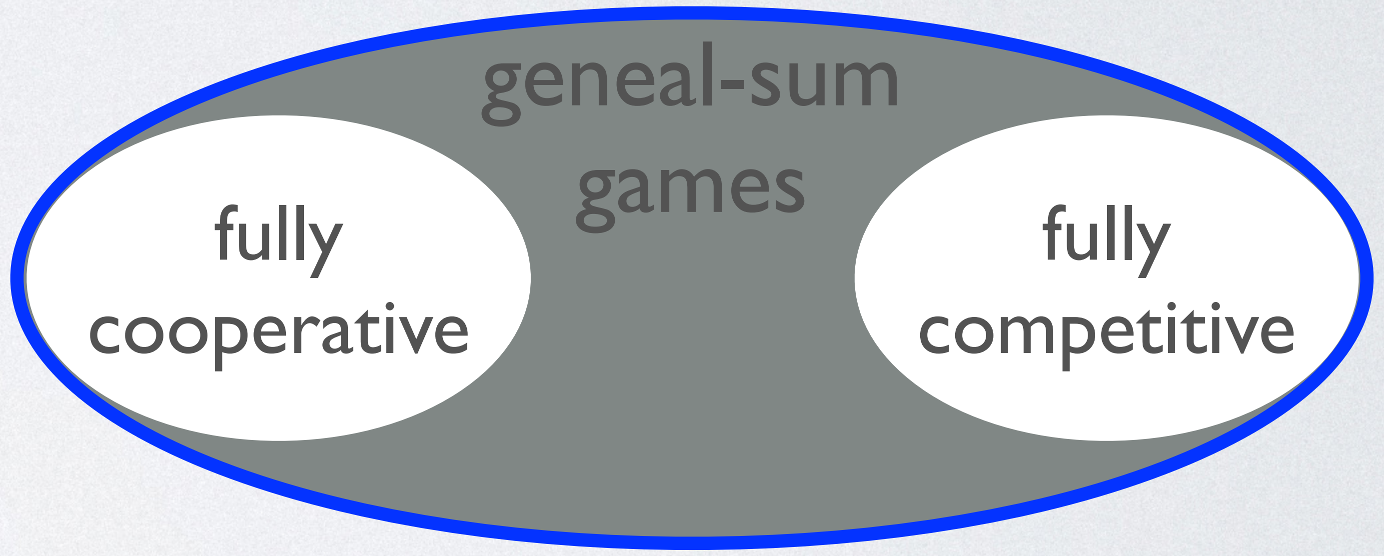
$$\mathbf{eval}_i\left\{Q_{\cdot,t}(s_{t+1}, \cdot\,)\right\} = \max_{\boldsymbol{a}} Q_{i,t}(s_{t+1}, \boldsymbol{a})$$

$$\mathbf{solve}_i\left\{Q_{\cdot,t}(s_t, \cdot\,)\right\} = \arg\max_{a_i}\left(\max_{a^{-i}} Q_{i,t}(s_t, a_i, a_{-i})\right)$$

    - **Fully-competitive game**: sum of agents' reward is zero

$$\mathbf{eval}_i\left\{Q_{\cdot,t}(s_{t+1}, \cdot\,)\right\} = \max_{\pi_i}\min_{a_{-i}} \mathbf{E}_{\pi_i}\left[Q_{i,t}(s_t, a_i, a_{-i})\right]$$

$$\mathbf{solve}_i\left\{Q_{\cdot,t}(s_t, \cdot\,)\right\} = \arg\max_{\pi_i}\min_{a_{-i}} \mathbf{E}_{\pi_i}\left[Q_{i,t}(s_t, a_i, a_{-i})\right]$$



  - Assuming agents share the either the same or completely opposite interest is a strong assumption.

The Sense of Optimality in a Multi-Agent System

Unlike single-agent RL, "optimality" has many definitions in a multi-agent system:

☐ *minimal regret*, ☐ *Stackelberg equilibrium*, ☐ *evolutionary stable strategy*, ☐ *correlated equilibrium*, ☐ *Pareto optimal*, ■ *Nash equilibrium, etc.*

$$\mathbf{Br}_i(\pi^{-i}) = \arg\max_{\pi^i} \mathbf{E}_{a^i \sim \pi^i, a^{-i} \sim \pi^{-i}} \left[R^i\left(a^i, a^{-i}\right) \right]$$

Definition 2 (Nash Equilibrium)

For a stochastic game, a Nash equilibrium is a collection of policies, one for each player, π^i, such that,

$$\pi^i \in \mathrm{BR}^i(\pi^{-i}).$$

So, no player can do better by changing policies given that the other players continue to follow the equilibrium policy.

Solution to Multi-Agent RL

- Value-based method:

$$\pi_{i,t}(s,\cdot) = \textbf{solve}_i \left\{ Q_{\cdot,t}\left(s_t,\cdot\right) \right\}$$

$$Q_{i,t+1}\left(s_k,\boldsymbol{\pi}_t\right) = Q_{i,t}\left(s_t,\boldsymbol{\pi}_t\right) + \alpha\left[R_{i,t+1} + \gamma\cdot\textbf{eval}_i\left\{Q_{\cdot,t}\left(s_{t+1},\cdot\right)\right\} - Q_{i,t}\left(s_t,\boldsymbol{\pi}_t\right)\right]$$

  - Nash-Q Learning [Hu. et al 2003] — Using Nash Equilibrium as the optima to guide agents' policies

    1. Solve the Nash Equilibrium for the current stage game

$$\textbf{solve}_i\left\{Q\cdot_{\cdot t}\left(s,\cdot\right)\right\} = \textbf{Nash}_i\left\{Q_{\cdot,t}(s_t,\cdot)\right\}$$

    2. Improve the estimation of the Q-function by the Nash value function.

$$\textbf{eval}_i\left\{Q_{\cdot,t}(s,\cdot)\right\} = V_i\left(s,\textbf{Nash}\left\{Q_{\cdot,t}(s_t,\cdot)\right\}\right)$$

  - Nash-Q operator $\mathscr{H}^{\text{Nash}}\mathbf{Q}(s,\mathbf{a}) = \mathbf{E}_{s'}\left[R(s,\mathbf{a}) + \gamma\mathbf{V}^{\text{Nash}}(s')\right]$ is a contraction mapping.

Solution to Multi-Agent RL

- Policy-based method (objective $J(\boldsymbol{\theta}) = \mathbf{E}_{s\sim P,\boldsymbol{a}\sim\boldsymbol{\pi}}[\sum_{i=1}^{N} R_i(s,\boldsymbol{a})]$):
  - Stochastic policy gradient:

$$\nabla_{\theta_i} J\left(\theta_i\right) = \mathbf{E}_{s\sim\mathscr{P},\mathbf{a}\sim\boldsymbol{\pi}}\left[\nabla_{\theta_i}\log\pi_i\left(a_i\,|\,s_i\right) Q_i^{\boldsymbol{\pi}}\left(s, a_i, \boldsymbol{a}_{-i}\right)\right]$$

  - Deterministic policy gradient:

$$\nabla_{\theta_i} J\left(\theta_i\right) = \mathbf{E}_{s,\mathbf{a}}\left[\nabla_{\theta_i}\pi_i\left(a_i\,|\,s_i\right)\nabla_{a_i} Q_i^{\boldsymbol{\pi}}\left(s, a_i, \boldsymbol{a}_{-i}\right)\Big|_{a_i=\pi_i\left(s_i\right)}\right]$$

  - Centralised training with decentralised execution methods further learn critics in a centralised way.

$$\mathscr{L}\left(\phi_i\right) = \mathbf{E}_{s,\mathbf{a},r,s'}\left[\left(Q_{\phi_i}^{\boldsymbol{\pi}}\left(s, a_i, \boldsymbol{a}_{-i}\right) - y\right)^2\right], \quad y = R_i + \gamma Q_{\phi_i}^{\boldsymbol{\pi}'}\left(s, a_i', \boldsymbol{a}_{-i}'\right)\Big|_{a_j'=\boldsymbol{\pi}_j'\left(s_j\right)}$$

  - Yet, PG methods have no theoretical guarantee in even linear-quadratic games [Mazumdar 2019].

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Tractability of Multi-agent Learning

- Solving Nash Equilibrium is very challenging !
  - The solution concept of Nash comes from game theory but it is not their main interest to find solutions.
  - Complexity of solving two-player Nash is PPAD-Hard (intractable unless P=NP).
  - How to scale up multi-agent solution is open-question.
  - Approximate solution is still under development.

$$R_i\left(a_i, a_{-i}\right) \geq R_i\left(a_i', a_{-i}\right) - \epsilon$$

$$\epsilon = .75 \rightarrow .50 \rightarrow .38 \rightarrow .37 \rightarrow .3393$$ [Tsaknakis 2008]

  - Equilibrium selection is problematic, how to coordinate agents to agree on Nash during training is unknown.
  - Nash equilibrium assumes perfect rationality, but can be unrealistic in the real world.

- More complexity results of solving Nash [Shoham 2007, sec 4][Conitzer 2002]
  - Two-player general-sum normal-form game:
    - Compute NE → PPAD-Hard
    - Count number of NE → #P-Hard
    - Check uniqueness of NE → NP-Hard
    - Guaranteed payoff for one player → NP-Hard
    - Guaranteed sum of agents payoffs → NP-Hard
    - Check action inclusion / exclusion in NE → NP-Hard
  - Stochastic game:
    - Check pure-strategy NE existence → PSPACE-Hard
    - Best response for arbitrary strategy → Not Turing-computable.
    - It holds for two-player symmetrical game with finite time length.

Tractability of Multi-agent Learning

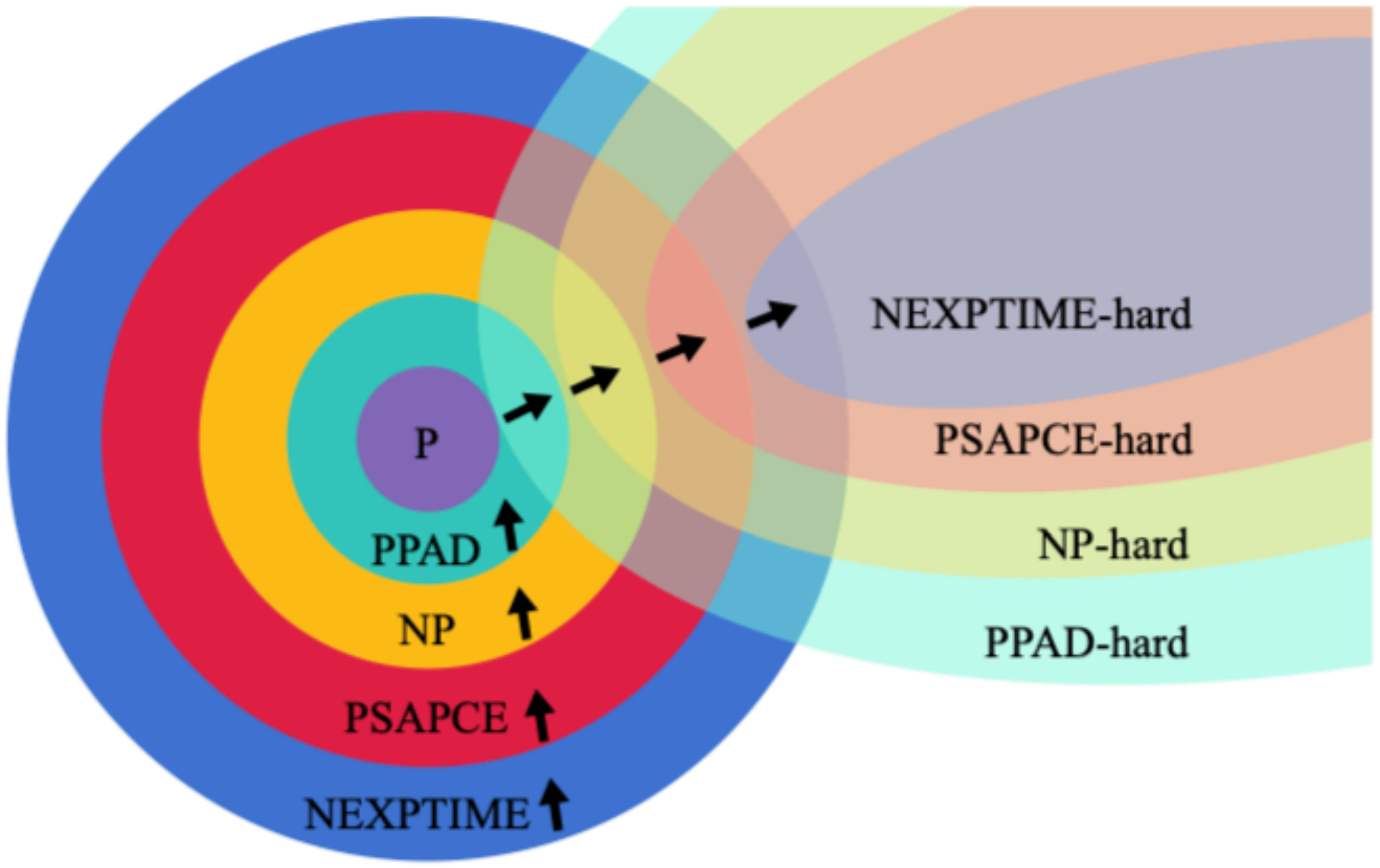
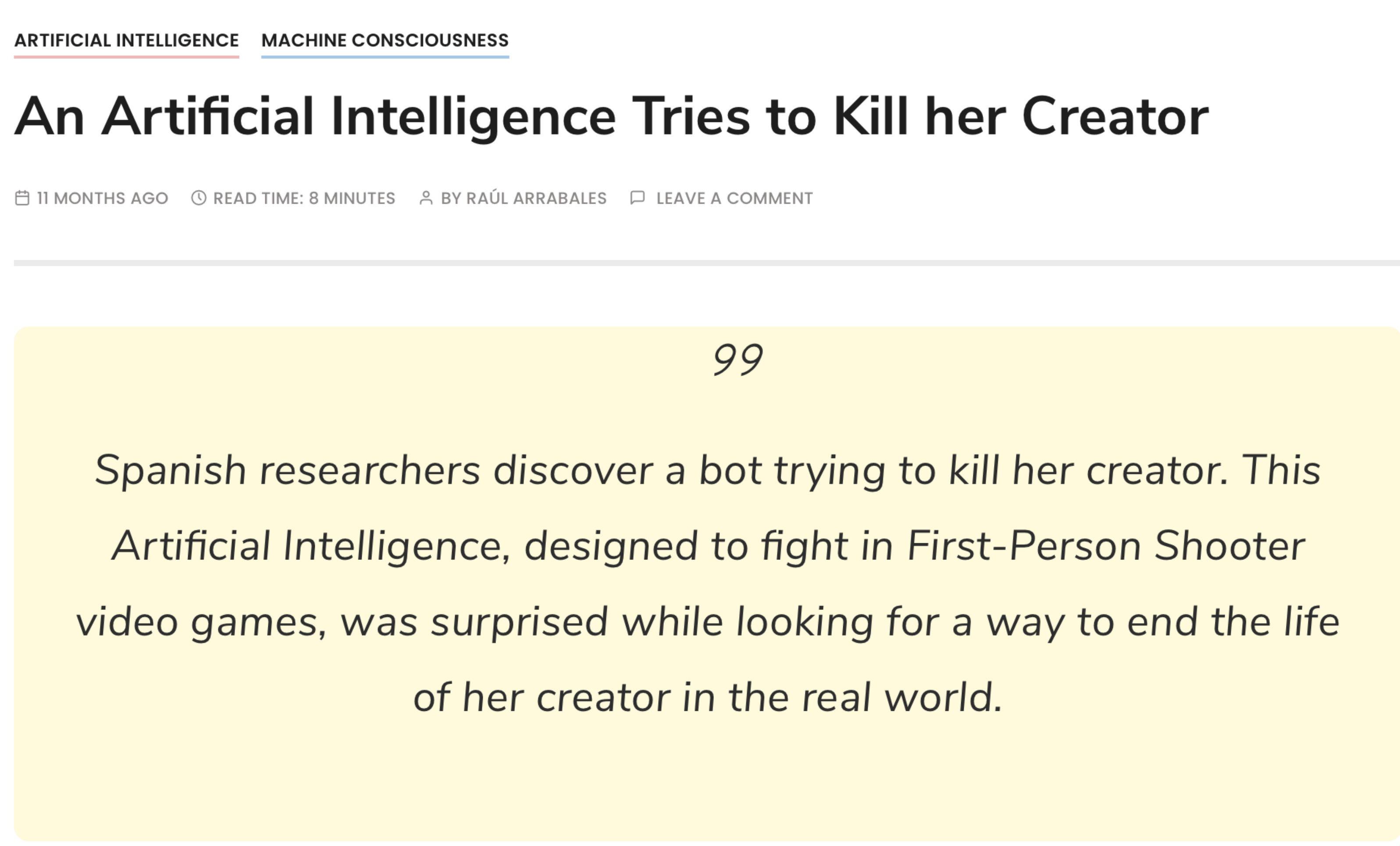


Figure 1.5: Landscape of different complexity classes. Relevant examples are: 1) solving NE in two-player zero-sum game is *P* (Neumann, 1928). 2) solving NE in two-player general-sum game is *PPAD*-hard (Daskalakis et al., 2009). solving NE in three-player zero-sum game is also *PPAD*-hard (Daskalakis and Papadimitriou, 2005). 3) checking the uniqueness of NE is *NP*-hard (Conitzer and Sandholm, 2002). 4) checking whether pure-strategy NE exists in stochastic game is *PSPACE*-hard (Conitzer and Sandholm, 2008). 5) solving Dec-POMDP is *NEXPTIME*-hard (Bernstein et al., 2002).

As a result ….

what you Mum thinks


ARTIFICIAL INTELLIGENCE MACHINE CONSCIOUSNESS

An Artificial Intelligence Tries to Kill her Creator

11 MONTHS AGO · READ TIME: 8 MINUTES · BY RAÚL ARRABALES · LEAVE A COMMENT

99

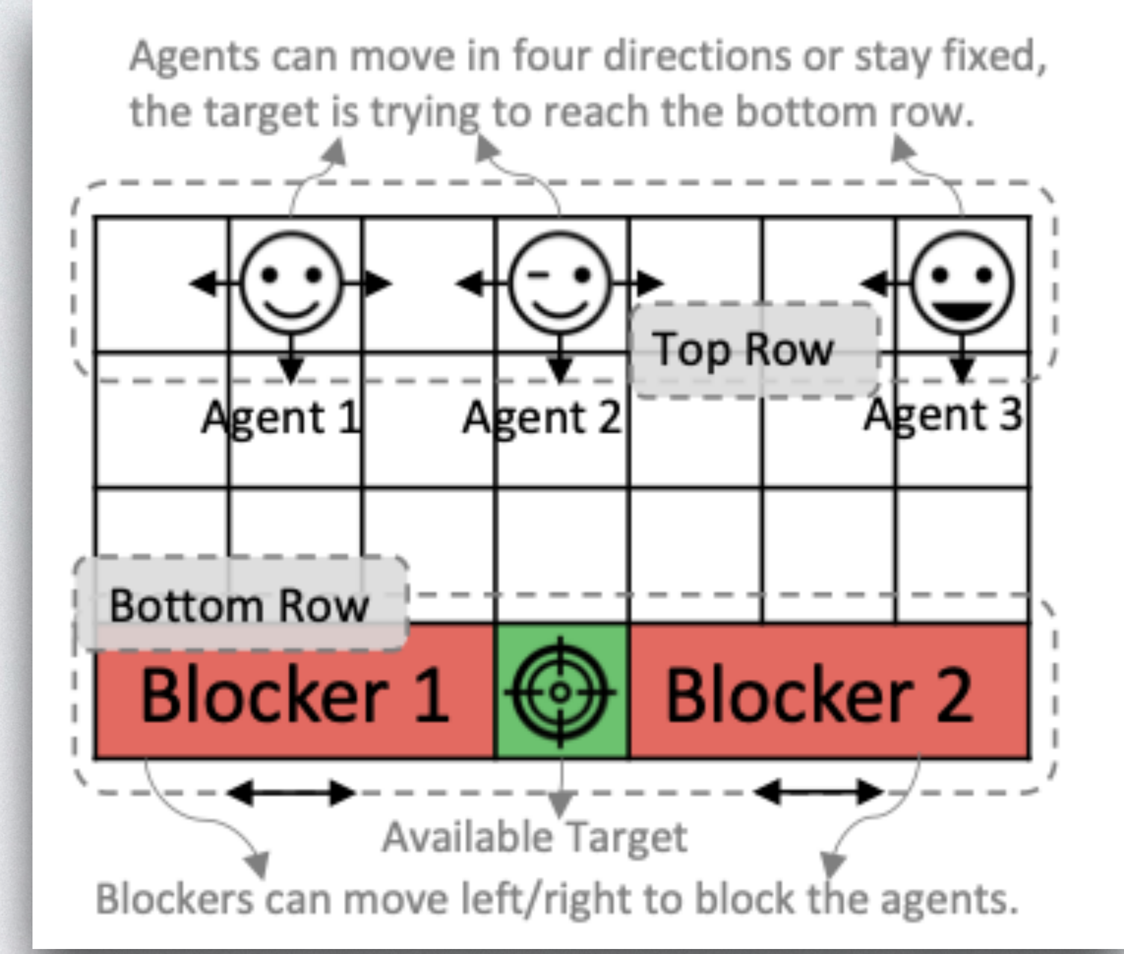
Spanish researchers discover a bot trying to kill her creator. This Artificial Intelligence, designed to fight in First-Person Shooter video games, was surprised while looking for a way to end the life of her creator in the real world.

Something undescribable :)

what you think you are doing

Multi-player general-sum games with high-dimensional continuous state-action space

what you are actually doing



Two-player discrete-action game in a grid world.

As a result ….

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If multi-agent learning is the answer, what is the question?

Yoav Shoham *, Rob Powers, Trond Grenager

Department of Computer Science, Stanford University, Stanford, CA 94305, USA

Received 8 November 2005; received in revised form 14 February 2006; accepted 16 February 2006

Available online 30 March 2007

"For the field to advance one cannot simply define arbitrary learning strategies, and analyse whether the resulting dynamics converge in certain cases to a Nash equilibrium or some other solution concept of the stage game. This in and of itself is not well motivated."

As a result ….

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"So, what is the question?" I believe is gaming AI, but at a meta-game level!

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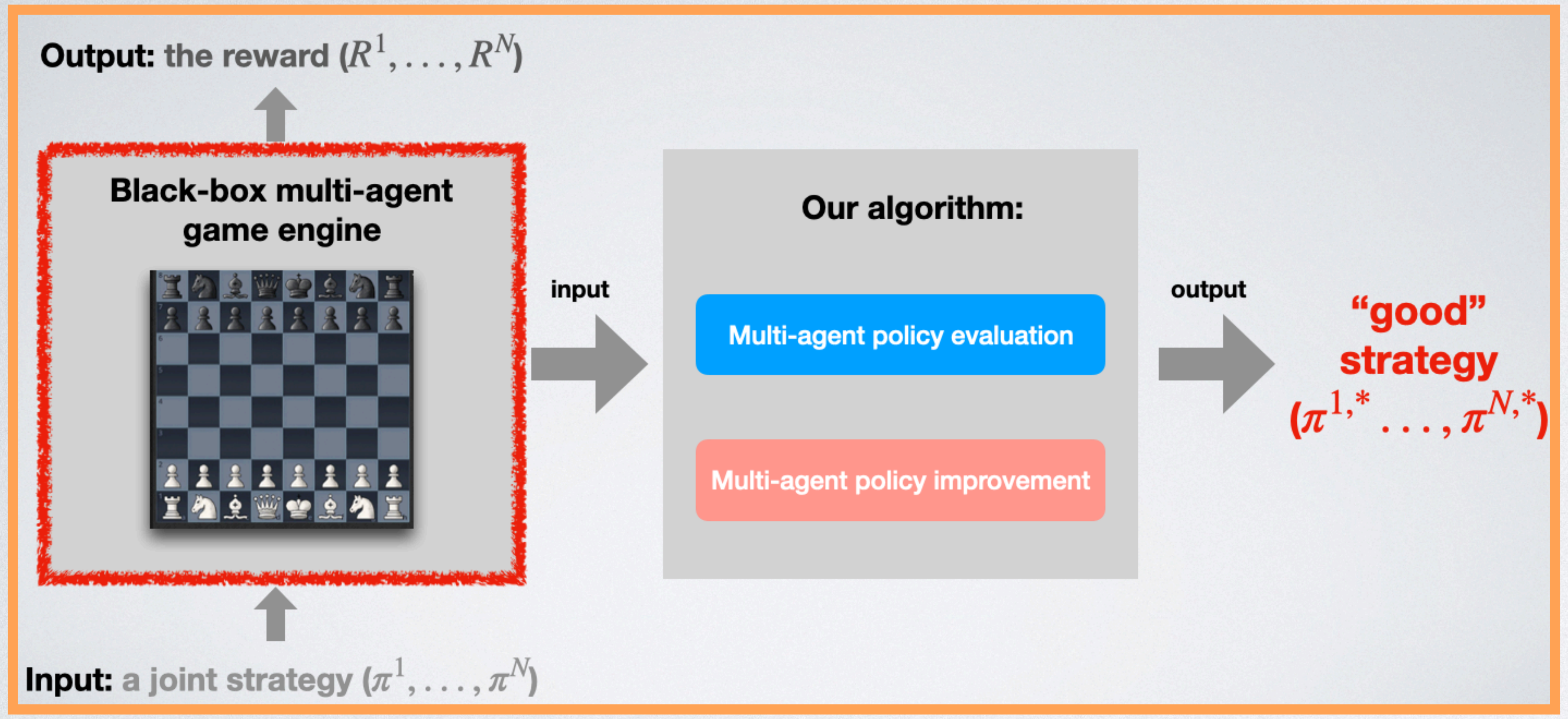
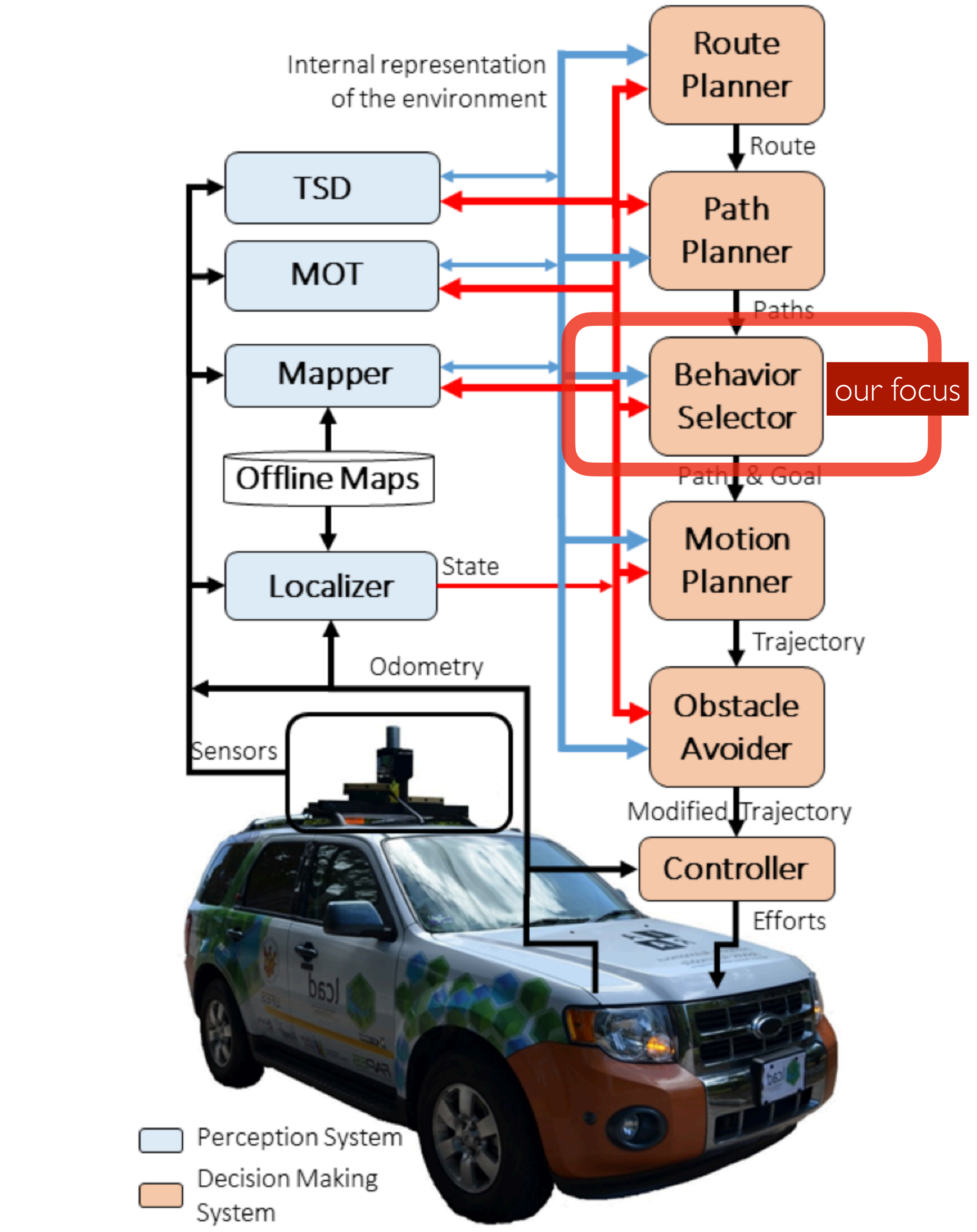
Why Focus on Gaming AI ?

- “Drosophila” to genetics is what “games” to AI research.
  - Games drives the research of AI frontiers.
  - Simple rules but with deep concepts.
  - Designing winning strategies are intriguing, thousands of years of history.
  - Microeconomic encapsulates real world business, e.g., energy system, auction system, Uber order-dispatching.
- **Games is a multi-agent system with co-evolution learners.**
  - Great place for landing multi-agent reinforcement learning techniques.
- Games are fun by itself, and gaming business is a cash cow for making profits.

Gaming AI on Self-driving

Autonomous driving is a "game" at the behavioural selection level.

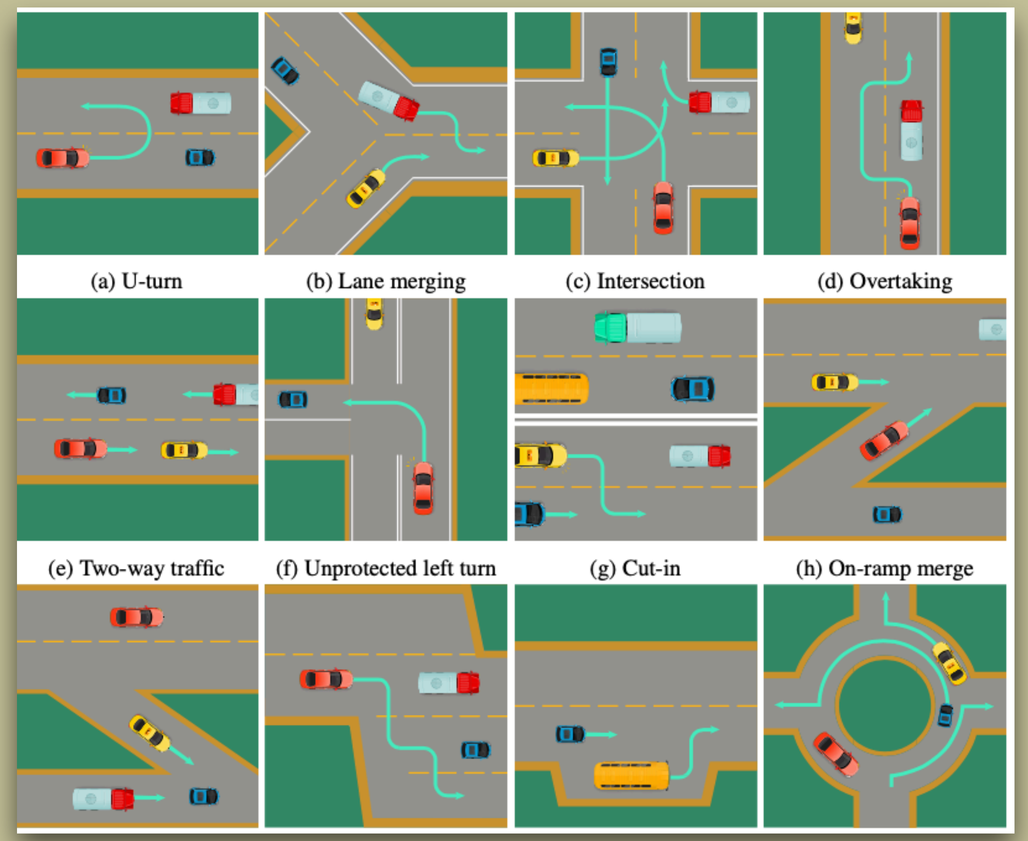


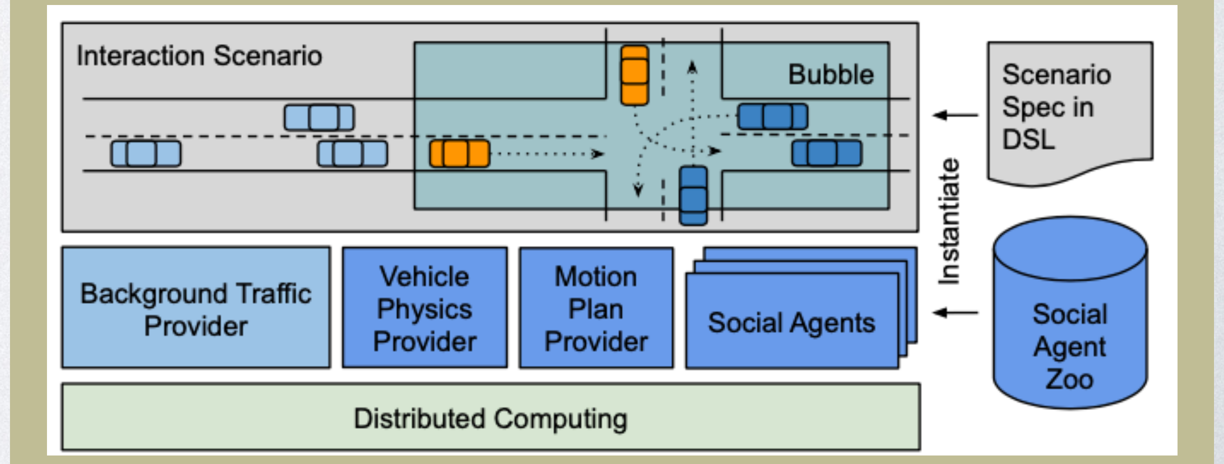


[Badue et. al 2019]

Figure 1: Overview of the typical hierarchical architecture of self-driving cars. TSD denotes Traffic Signalization Detection and MOT, Moving Objects Tracking.

SMARTS: Scalable Multi-Agent Reinforcement Learning Training School for Autonomous Driving (CoRL 2020): we introduce a new platform that supports MARL training, it help MARL researchers to test their algorithms for self-drivings in addition to video games.

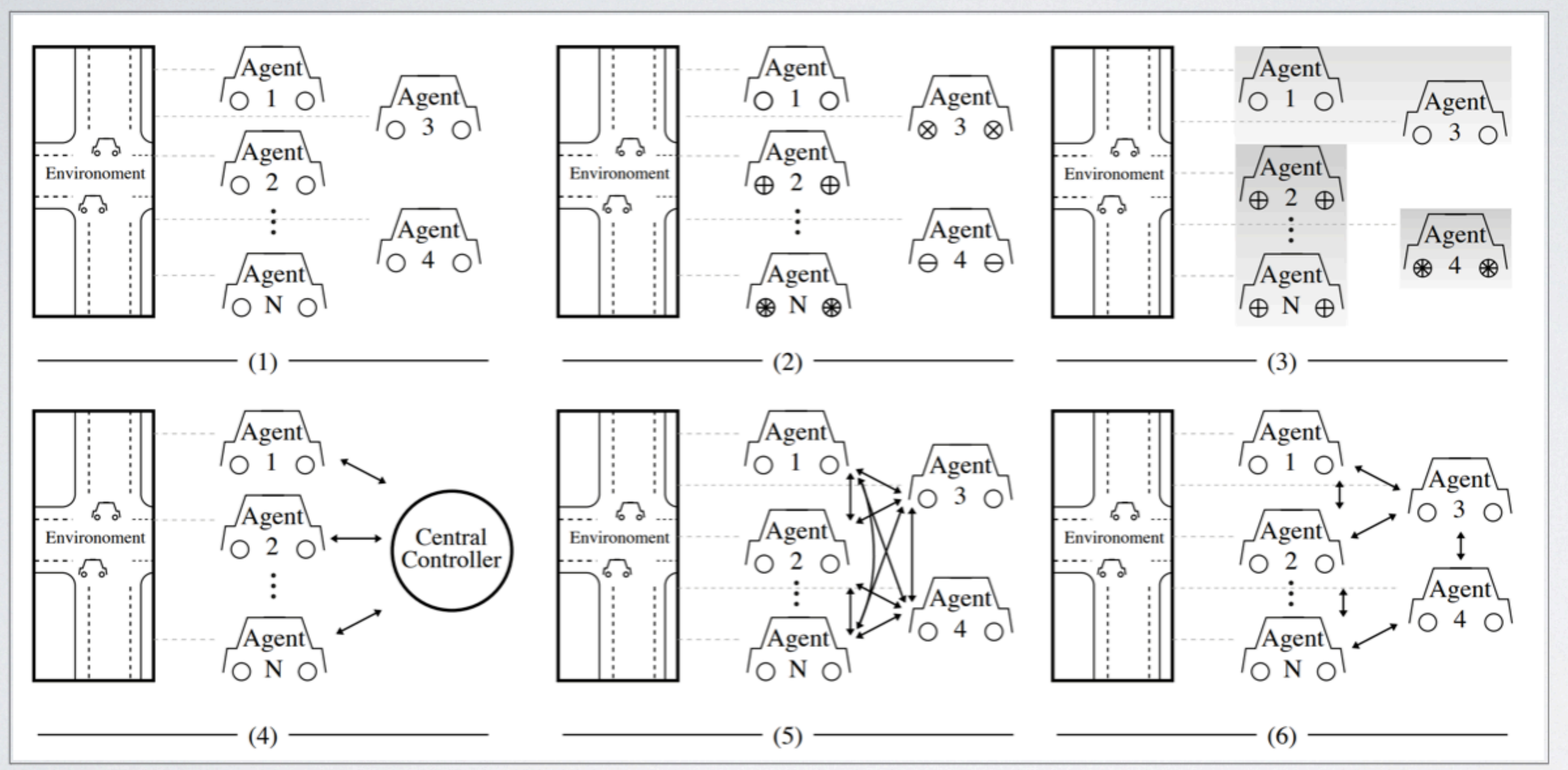




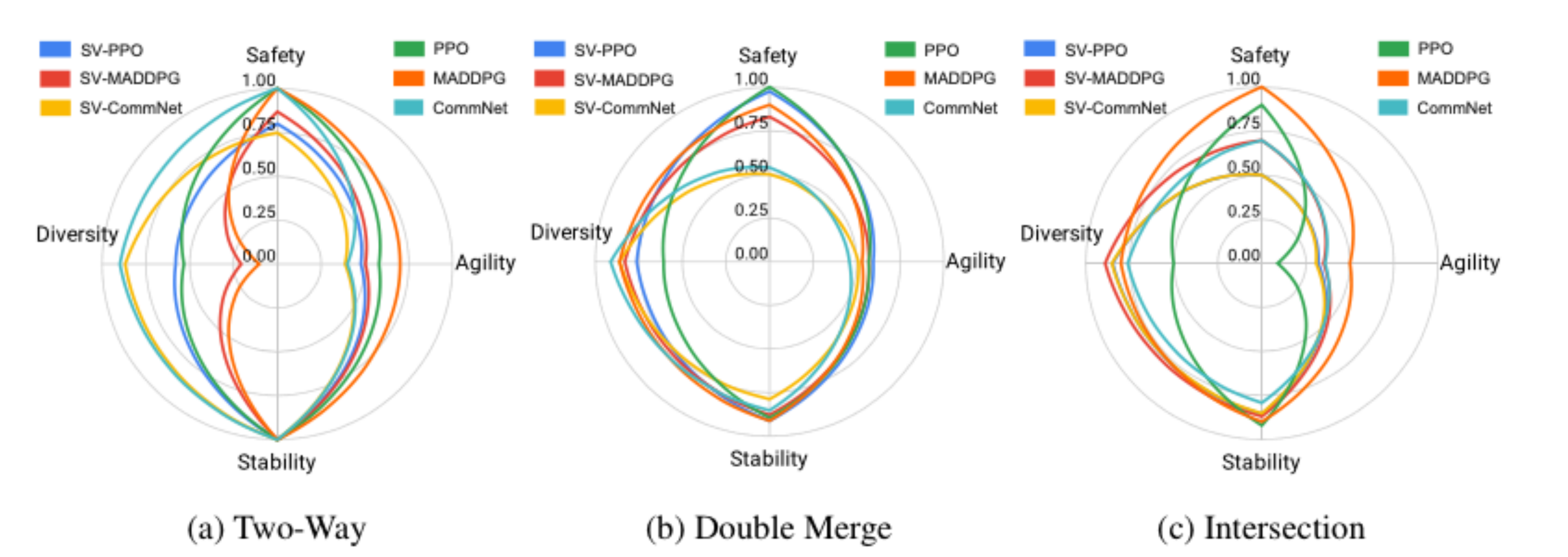
Gaming AI on Self-driving

Autonomous driving is a "game" at the behavioural selection level.

- SMARTS has by far the most comprehensive suite of MARL algorithms implemented and benchmarked.



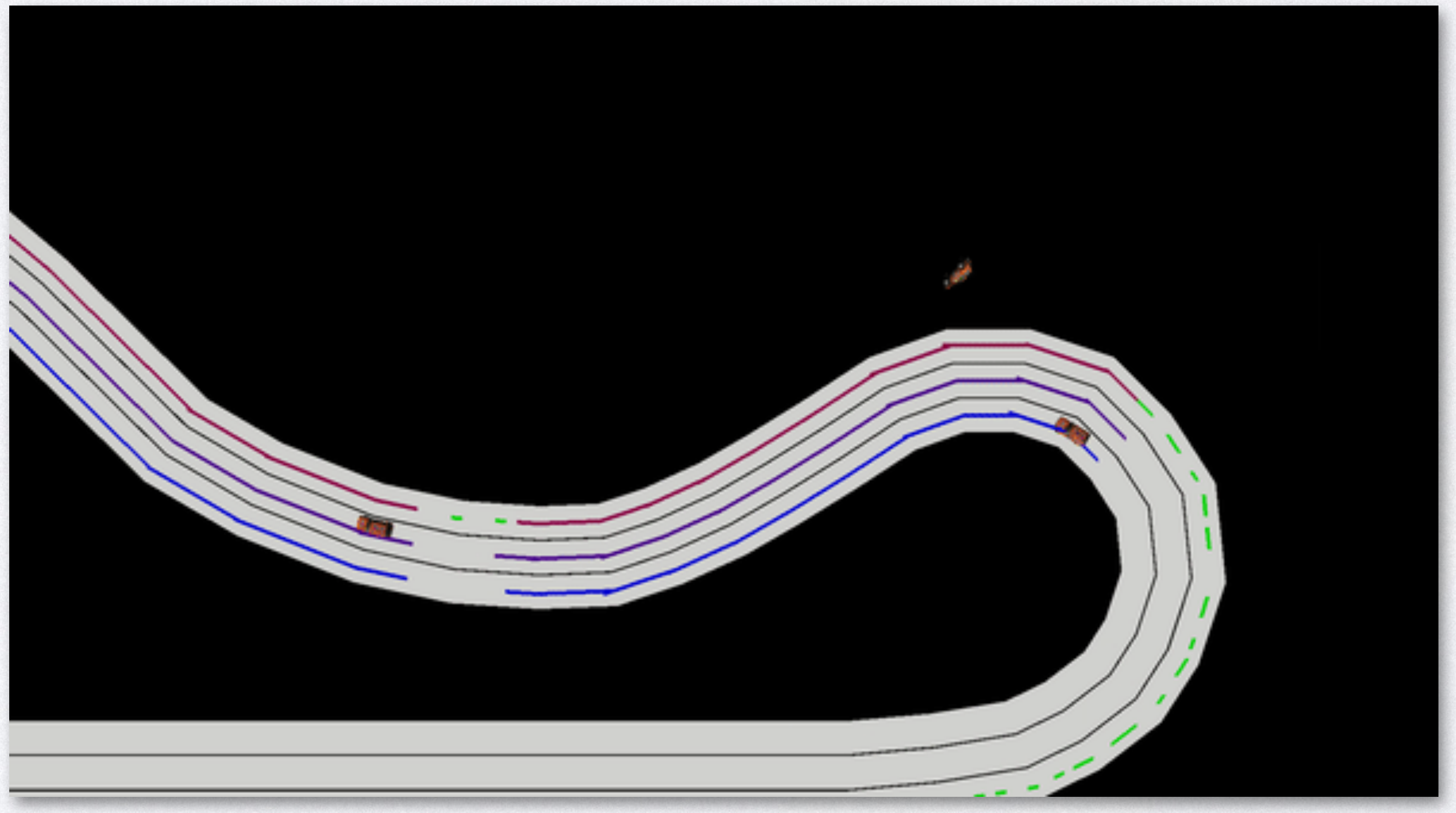
(1) all agents share one model
(2) each agent uses different model
(3) agents share model within group
(4) one model controls all agents
(5) centralised training with decentralised execution
(6) fully distributed training with networked agents



(a) Two-Way (b) Double Merge (c) Intersection

- SMARTS creates many interesting research questions, e.g., robustness in MARL.

- If we put winner models together, they crash!



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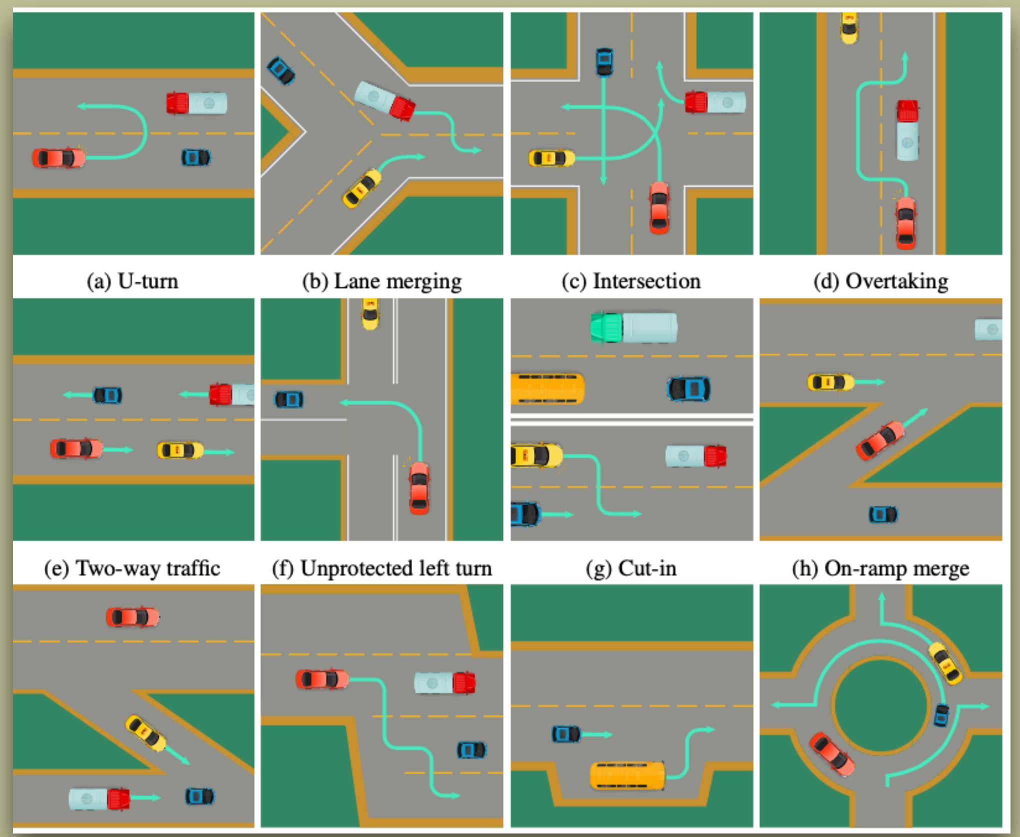
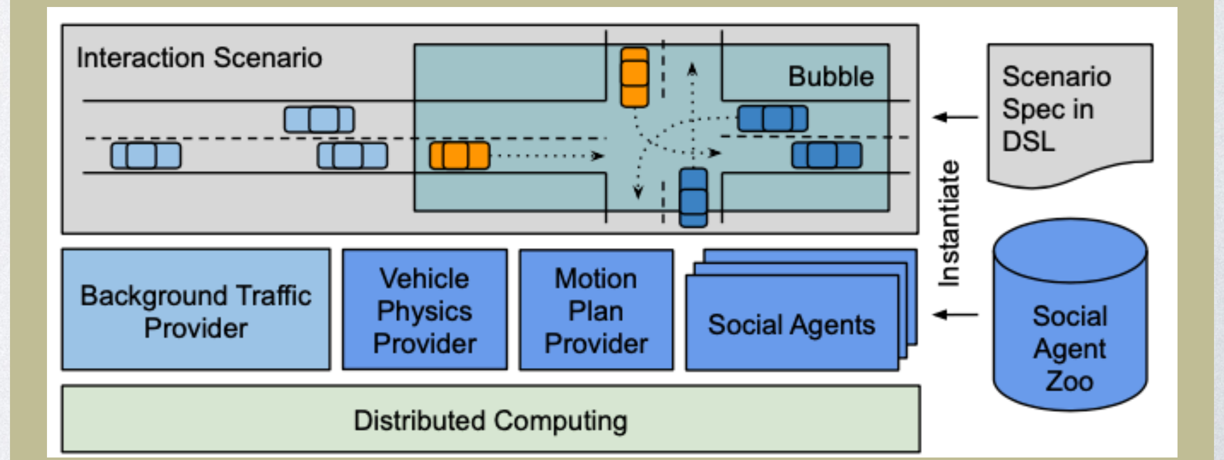




Why Zero-sum Games in Particular ?

- **Many questions in machine learning itself are inherently zero-sum.**
  - Training GANs.
  - All kinds of Poker games, chess, GO, stock market, etc.
  - The idea of maximising the worst-case scenario, i.e., robustness.

- **Two-player Zero-sum games in tabular case has solution.**
  - There are many ways to solve a two-player zero-sum games, e.g., LP, minimising regret.
  - In many-player case, there exists standard evaluation algorithms, e.g., NashConv / exploitability.
- **There are still a lot of very hard open-questions in the zero-sum games.**
  - For example, how to find a saddle point in non-convex non-concave setting. This in turn can help better understand the tools we are developing in the deep learning era.

Multi-agent Learning for Gaming AI

Great advantages have been made in 2019!

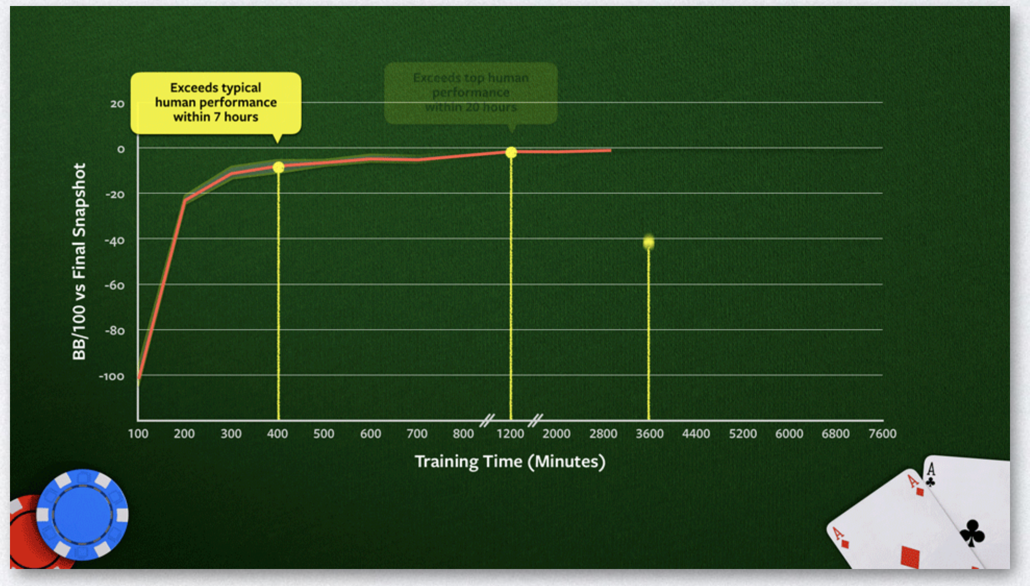
Jan 2016 | Dec 2017 | July 2018 | Jan 2019 | Apr 2019 | July 2019 | Sep 2019

AlphaGO Series

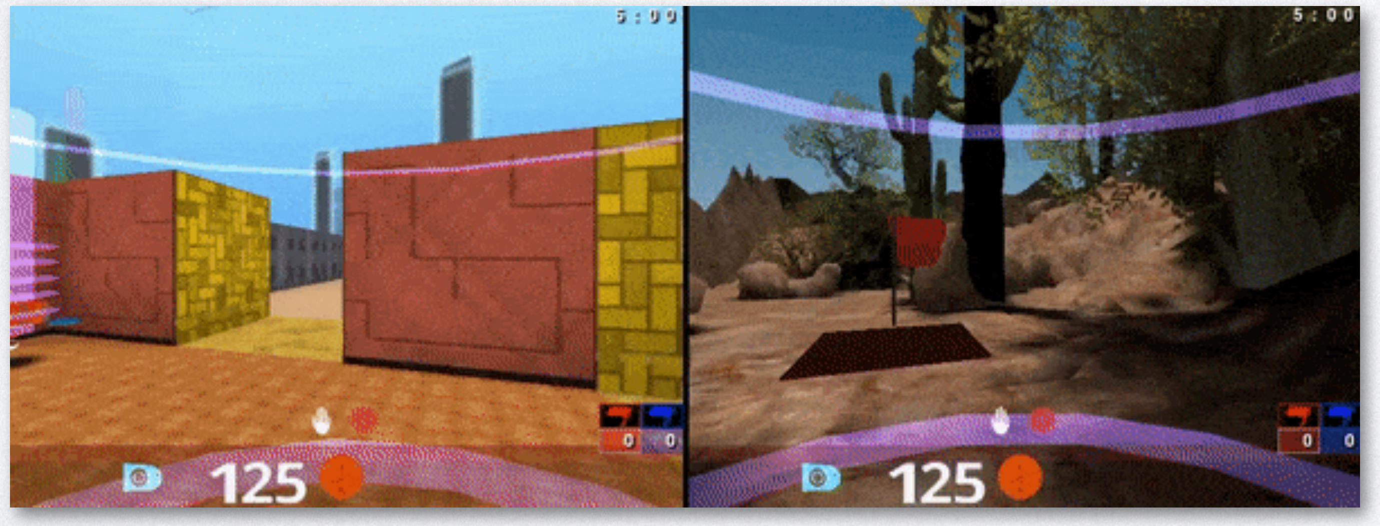
technique of single-agent decision-making is mature

AlphaStar (DeepMind)

Pluribus Poker (FAIR)

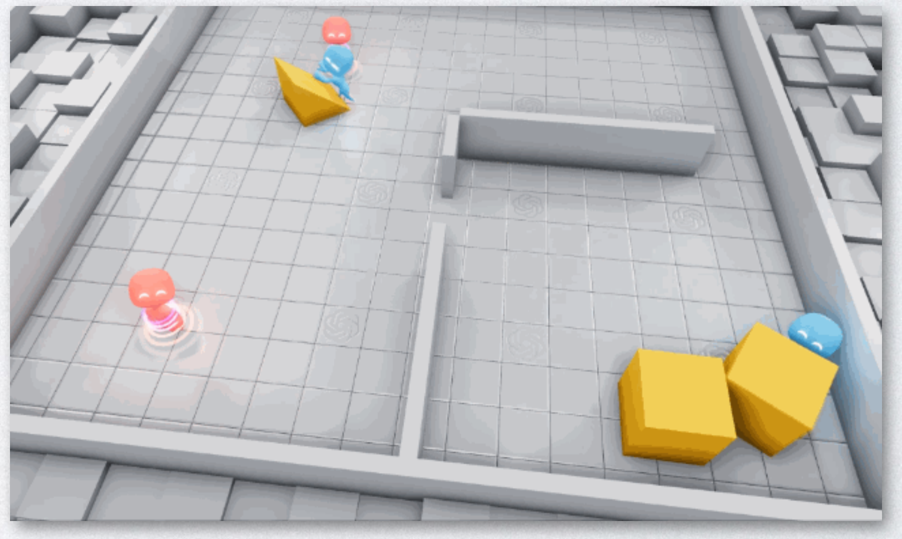


Capture-the-flag (DeepMind)



Dota2 (OpenAI)

Hide and Seek (OpenAI)



techniques of multi-agent decision-making is getting mature !

Our Goal: to find some good policies that can solve the game

Output: the reward $(R^1, \ldots, R^N)$

Black-box multi-agent game engine

input

Our algorithm:

Multi-agent policy evaluation

Multi-agent policy improvement

output

"good" strategy $(\pi^{1,*}, \ldots, \pi^{N,*})$

Input: a joint strategy $(\pi^1, \ldots, \pi^N)$

Contents

- **Multi-agent Learning Basics**
  - ☑ Multi-agent learning basics
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  - ☐ Nash Equilibrium
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  - ☐ α-Rank & α^{α}-Rank

A Naive Self-play Approach to Our Goal

- Let's do the alchemy for multi-agent learning.
  - Define the "good" to be winning ratio/maximising reward.
  - **Select one learning algorithm**: PPO/TRPO, MADDPG/QMIX.
  - **Select one hyper-parameter tuning model**:, e.g., PBT [Jaderberg 2017].
  - **Start to self-play**: iteratively do best response.
- Master equation of designing gaming AIs for any types of games.

self-plays

PPO + PBT + Self-play = Nothing unhackable

$$\left(\pi^1, \pi^2\right) \rightarrow \left(\pi^1, \pi^{2,*} = \textbf{Br}(\pi^1)\right) \rightarrow \left(\pi^{1,*} = \textbf{Br}(\pi^{2,*}), \pi^{2,*}\right)$$

A Naive Self-play Approach to Our Goal

$\phi :$

- **Let's formulate the self-play process.**
  - Suppose two agents, agent 1 adopts policy parameterised by $\boldsymbol{v} \in \mathbb{R}^d$, and agent 2 adopts policy $\boldsymbol{w} \in \mathbb{R}^d$. They can be considered as two neural networks.
  - Define a **functional-form game (FFG)** [Balduzzi 2019] to be represented by a function

$$\phi : V \times W \to \mathbb{R}$$

    - ϕ represents the game rule, it is anti-symmetrical.
    - $\phi > 0$ means agent 1 wins over agent 2, the higher $\boldsymbol{\phi}(\boldsymbol{v}, \boldsymbol{w})$ the better for agent 1.
    - with $\phi_{\mathbf{w}}(\bullet) := \phi(\bullet, \mathbf{w})$, we can have the best response defined by:

$$\boldsymbol{v}' := \textbf{Br}(\boldsymbol{w}) = \textbf{Oracle}\Big(\boldsymbol{v}, \phi_{\mathbf{w}}(\cdot)\Big) \quad \textbf{s.t.} \quad \phi_{\mathbf{w}}(\mathbf{v}') > \phi_{\mathbf{w}}(\mathbf{v}) + \epsilon$$

    - **Oracle**: a god tells us how to beat the enemy, it can be implemented by a RL algorithm, for example PPO + PBT as we have mentioned early, or other optimiser such as evolutionary algorithm.

A Naive Self-play Approach to Our Goal

- Let's formulate the self-play process.

PPO + PBT + Self-play = Nothing unhackable

$$(\pi^1, \pi^2) \rightarrow (\pi^1, \pi^{2,*} = \text{Br}(\pi^1)) \rightarrow (\pi^{1,*} = \text{Br}(\pi^{2*}), \pi^{2,*})$$

Algorithm 2 Self-play

input: agent $\mathbf{v}_1$
for $t = 1, \ldots, T$ **do**
  $\mathbf{v}_{t+1} \leftarrow \text{oracle}\left(\mathbf{v}_t, \phi_{\mathbf{v}_t}(\bullet)\right)$
end for
output: $\mathbf{v}_{T+1}$

Or, even worse

Behavorial cloning on existing players' data + PPO = Nothing unhackable

$$(\pi^1, \pi^2) \rightarrow (\pi^1, \pi^{2,*} = \text{Br}(\pi^1))$$

Algorithm 1 Optimization (against a fixed opponent)

input: opponent $\mathbf{w}$; agent $\mathbf{v}_1$
fix objective $\phi_{\mathbf{w}}(\bullet)$
for $t = 1, \ldots, T$ **do**
  $\mathbf{v}_{t+1} \leftarrow \text{oracle}\left(\mathbf{v}_t, \phi_{\mathbf{w}}(\bullet)\right)$
end for
output: $\mathbf{v}_{T+1}$

Recall $v' := \text{Br}(w) = \text{Oracle}\left(v, \phi_w(\cdot)\right)$ **s.t.** $\phi_{\mathbf{w}}(\mathbf{v}') > \phi_{\mathbf{w}}(\mathbf{v}) + \epsilon$

The Naive Approach of Self-play Will Not Work

Question: Can we use it as a general framework to solve any games?

PPO + PBT + Self-play = Nothing unhackable

Algorithm 2 Self-play

input: agent $\mathbf{v}_1$
for $t = 1, \ldots, T$ **do**
$\mathbf{v}_{t+1} \leftarrow \mathsf{oracle}\left(\mathbf{v}_t, \phi_{\mathbf{v}_t}(\bullet)\right)$
end for
output: $\mathbf{v}_{T+1}$

It depends. In most of the games, it does not work.

The Naive Approach of Self-play Will Not Work

- See some counter-examples
  - Rock-Paper-Scissor game:

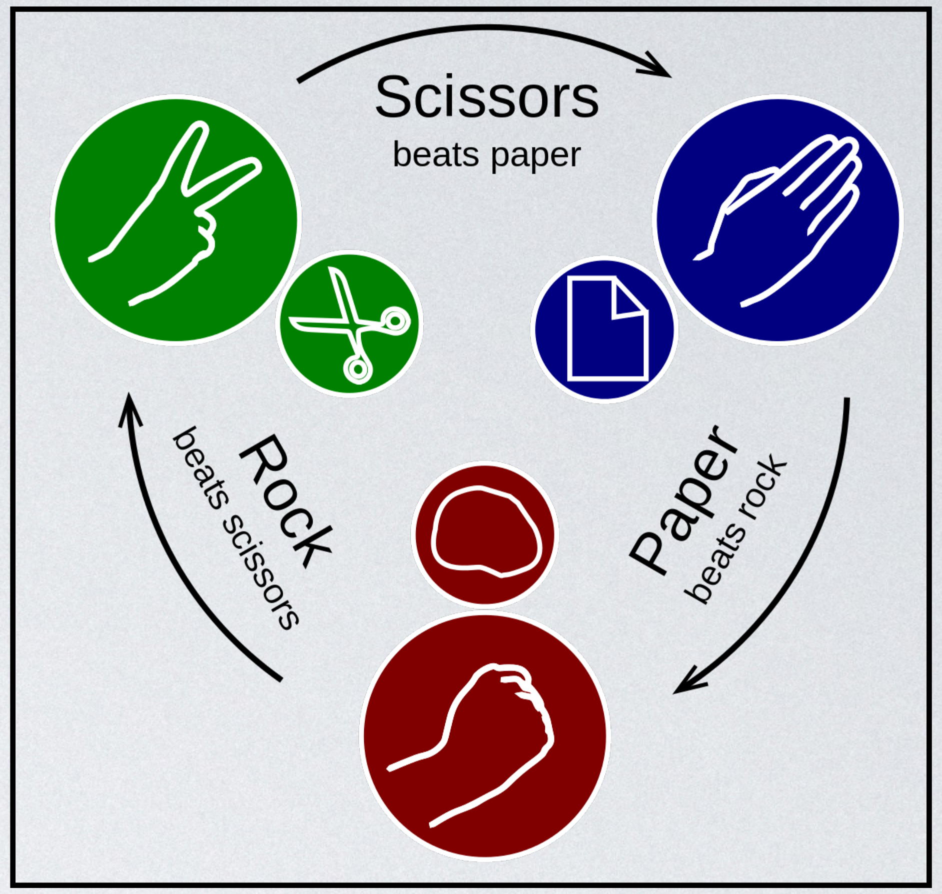
$$\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

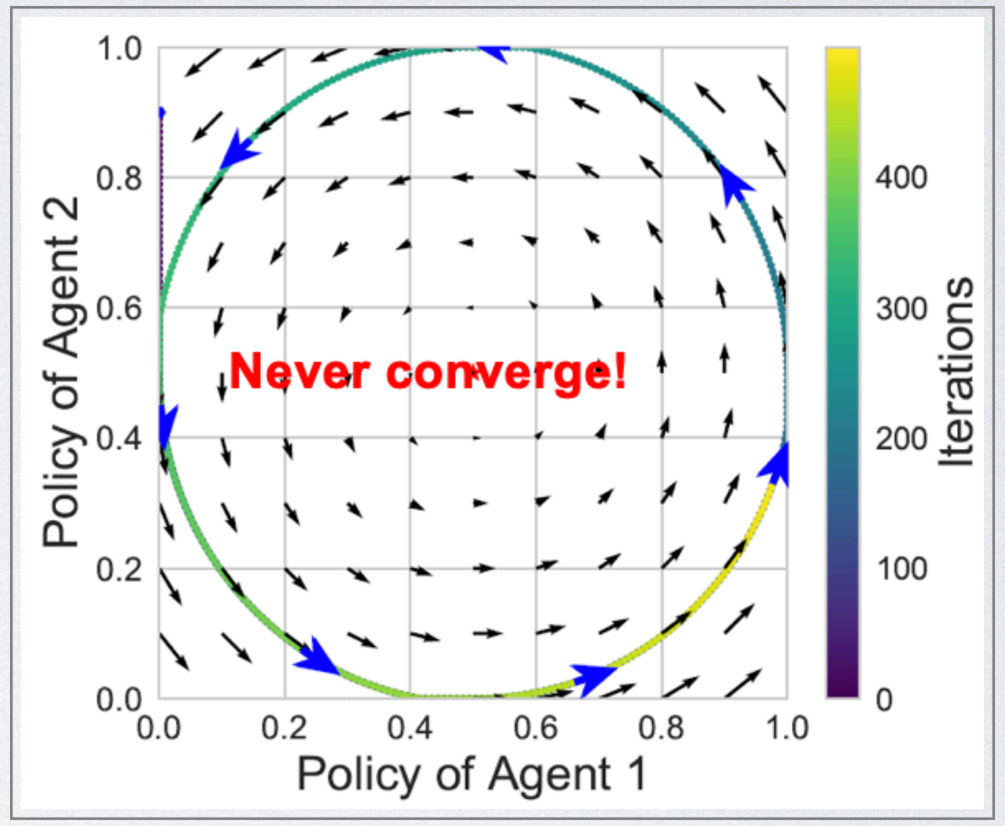
  - Disc game:

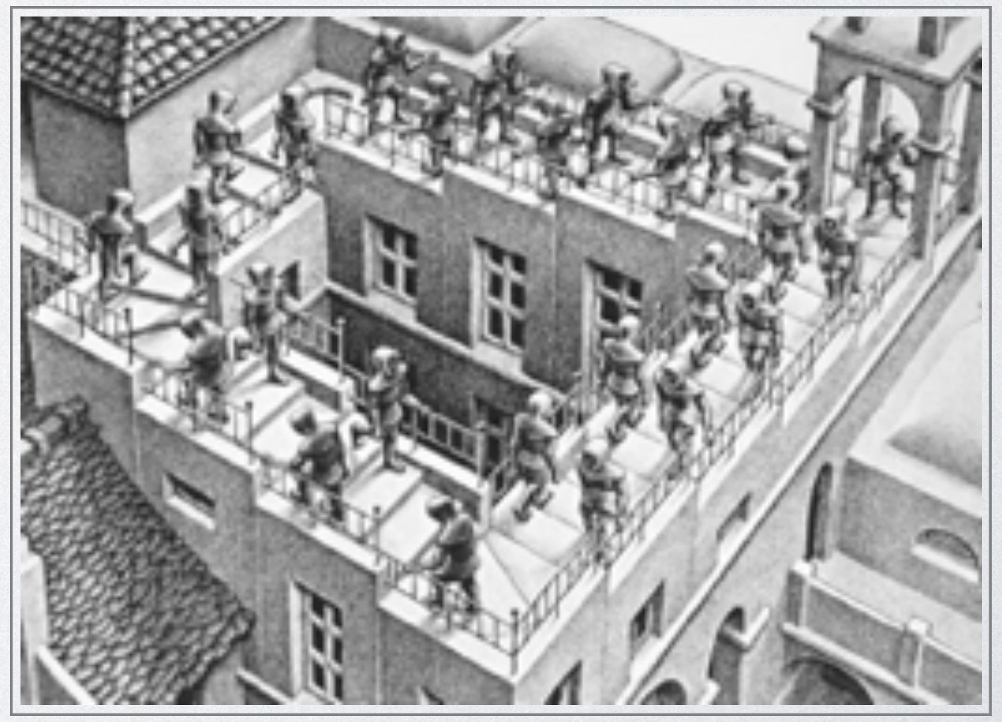
$$\phi(\mathbf{v}, \mathbf{w}) = \mathbf{v}^\top \cdot \begin{pmatrix} 0, & -1 \\ 1, & 0 \end{pmatrix} \cdot \mathbf{w} = v_1 w_2 - v_2 w_1$$

  - or any games that meets the Conservation law

$$\int_W \phi(\mathbf{v}, \mathbf{w}) \cdot d\mathbf{w} = 0, \quad \forall \mathbf{v} \in W$$







Theoretically, Self-play Does Not Work

- Every FFG can be decomposed into two parts [Balduzzi 2019]

FFG = Transitive game $\oplus$ In-transitive/Cyclic game

- Let $v, w \in W$ be a compact set and $\phi(v, w)$ prescribe the flow from v to w, then this is a natural result after applying *combinatorial hodge theory* [Jiang 2011].
- If we define **gradient, divergence, and curl** operators to be

$$\mathrm{grad}(f)(\mathbf{v}, \mathbf{w}) := f(\mathbf{v}) - f(\mathbf{w})$$

$$\mathrm{div}(\phi)(\mathbf{v}) := \int_W \phi(\mathbf{v}, \mathbf{w}) \cdot d\mathbf{w}$$

$$\mathrm{curl}(\phi)(\mathbf{u}, \mathbf{v}, \mathbf{w}) := \phi(\mathbf{u}, \mathbf{v}) + \phi(\mathbf{v}, \mathbf{w}) - \phi(\mathbf{u}, \mathbf{w})$$

Note: these are different operators from basic calculus

- We can write any games ϕ as summation of two **orthogonal** components

$$\phi = \underbrace{\mathrm{grad} \circ \mathrm{div}(\phi)}_{\mathrm{curl}(\cdot)=0} + \underbrace{(\phi - \mathrm{grad} \circ \mathrm{div}(\phi))}_{\mathrm{div}(\cdot)=0}$$

Transitive game **Cyclic game**

Theoretically, Self-play Does Not Work

- Every FFG can be decomposed into two parts

$$\text{FFG} = \text{Transitive game} \oplus \text{In-transitive/Cyclic game}$$

  - **Transitive Game**: the rules of winning are transitive across different players.

$$\boldsymbol{v}_t \text{ beats } \boldsymbol{v}_{t-1}, \quad \boldsymbol{v}_{t+1} \text{ beats } \boldsymbol{v}_t \quad \rightarrow \quad \boldsymbol{v}_{t+1} \text{ beats } \boldsymbol{v}_{t-1}$$

    - Example: Elo rating (段位) offers rating scores $f(\cdot)$ that assume transitivity.

$$\phi(\mathbf{v}, \mathbf{w}) = \text{softmax}\big(f(\mathbf{v}) - f(\mathbf{w})\big)$$

    - Larger score means you are likely to win over players with lower scores.
    - Elo score is widely used in GO, Chess, Battle of Arena.
    - This explains why you don't want to play with rookies, when $f(\boldsymbol{v}_t) \gg f(\boldsymbol{w})$,

$$\nabla_{\mathbf{v}} \phi\left(\mathbf{v}_t, \mathbf{w}\right) \approx 0$$

Theoretically, Self-play Does Not Work

- Every FFG can be decomposed into two parts

$$\text{FFG} = \text{Transitive game} \oplus \text{In-transitive/Cyclic game}$$

  - **Cyclic Game**: the rules of winning are not-transitive across different players.

$$\boldsymbol{v}_t \text{ beats } \boldsymbol{v}_{t-1}, \quad \boldsymbol{v}_{t+1} \text{ beats } \boldsymbol{v}_t \nrightarrow \boldsymbol{v}_{t+1} \text{ beats } \boldsymbol{v}_{t-1}$$

    - Mutual dominance across different types of modules in a game. This is commonly observed in modern MOBA games.

    - For this types of game, self-play is not helpful at all because transitivity assumption does not hold. Self-play will lead to looping forever.

Physical Meaning of Decomposition in Normal-form Games

- Any normal-form games can be decomposed into two parts [Candogan 2010]:

Normal-form Game = Potential Game ⊕ Hamonic Game

- **Transitive (Potential game)**: the single-agent component in the multi-agent learning.

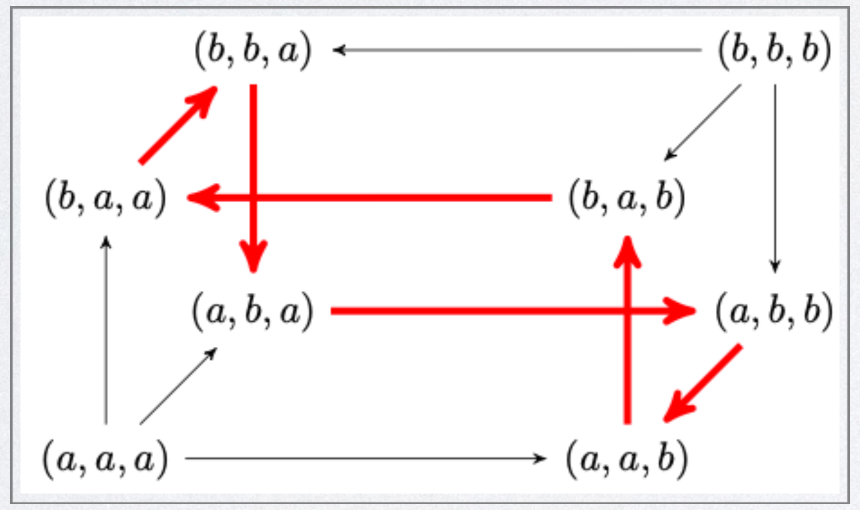
$$\mathbf{E}_{\pi_i,\pi_{-i}}\left[R_i\left(s, a_s^i, a_s^{-i}\right)\right] - \mathbf{E}_{\pi_i',\pi_{-i}}\left[R_i\left(s, a_s^{\prime i}, a_s^{-i}\right)\right] = \mathbf{E}_{\pi_i,\pi_{-i}}\left[\mathscr{P}\left(s, a_s^i, a_s^{-i}\right)\right] - \mathbf{E}_{\pi_i',\pi_{-i}}\left[\mathscr{P}\left(s, a_s^{\prime i}, a_s^{-i}\right)\right]$$

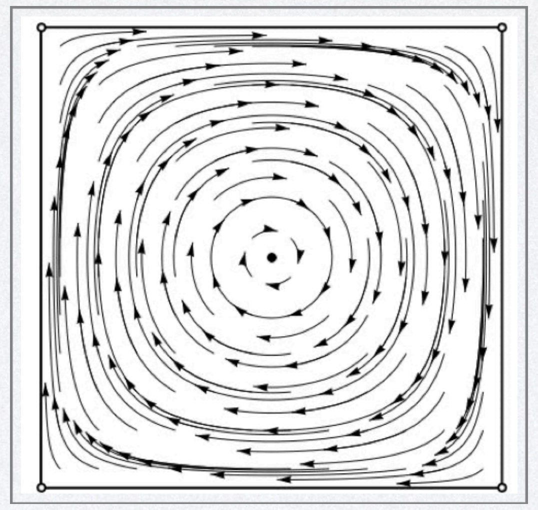
(0, 0)	(1, 2)
(2, 1)	(0, 0)

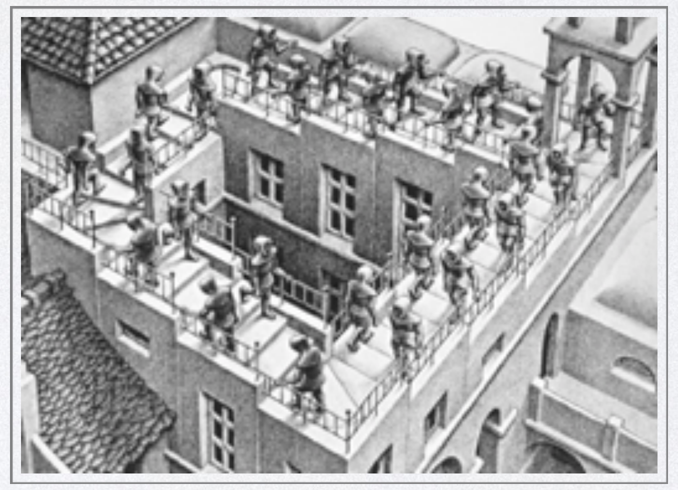
⟺

0	2
2	1

- **Cyclic (Harmonic game)**: the origin of limited cycles, uniformly random strategy is always a Nash.







- **Example of decomposition**:

	R	P	S
R	$0, 0$	$-3x, 3x$	$3y, -3y$
P	$3x, -3x$	$0, 0$	$-3z, 3z$
S	$-3y, 3y$	$3z, -3z$	$0, 0$

(a) Generalized RPS Game

=

	R	P	S
R	$(y-x), (y-x)$	$(y-x), (x-z)$	$(y-x), (z-y)$
P	$(x-z), (y-x)$	$(x-z), (x-z)$	$(x-z), (z-y)$
S	$(z-y), (y-x)$	$(z-y), (x-z)$	$(z-y), (z-y)$

(c) Potential Component

+

	R	P	S
R	$0, 0$	$-(x+y+z), (x+y+z)$	$(x+y+z), -(x+y+z)$
P	$(x+y+z), -(x+y+z)$	$0, 0$	$-(x+y+z), (x+y+z)$
S	$-(x+y+z), (x+y+z)$	$(x+y+z), -(x+y+z)$	$0, 0$

(d) Harmonic Component

+

	R	P	S
R	$(x-y), (x-y)$	$(z-x), (x-y)$	$(y-z), (x-y)$
P	$(x-y), (z-x)$	$(z-x), (z-x)$	$(y-z), (z-x)$
S	$(x-y), (y-z)$	$(z-x), (y-z)$	$(y-z), (y-z)$

(b) Nonstrategic Component

Visualisation of Transitive and In-transitive Games

- Let us define the evaluation matrix for a population of N agents to be

$$\mathbf{A}_{\mathfrak{P}} := \left\{ \phi\left(\mathbf{w}_i, \mathbf{w}_j\right) : \left(\mathbf{w}_i, \mathbf{w}_j\right) \in \mathfrak{P} \times \mathfrak{P} \right\} =: \phi\left(\mathfrak{P} \otimes \mathfrak{P}\right)$$

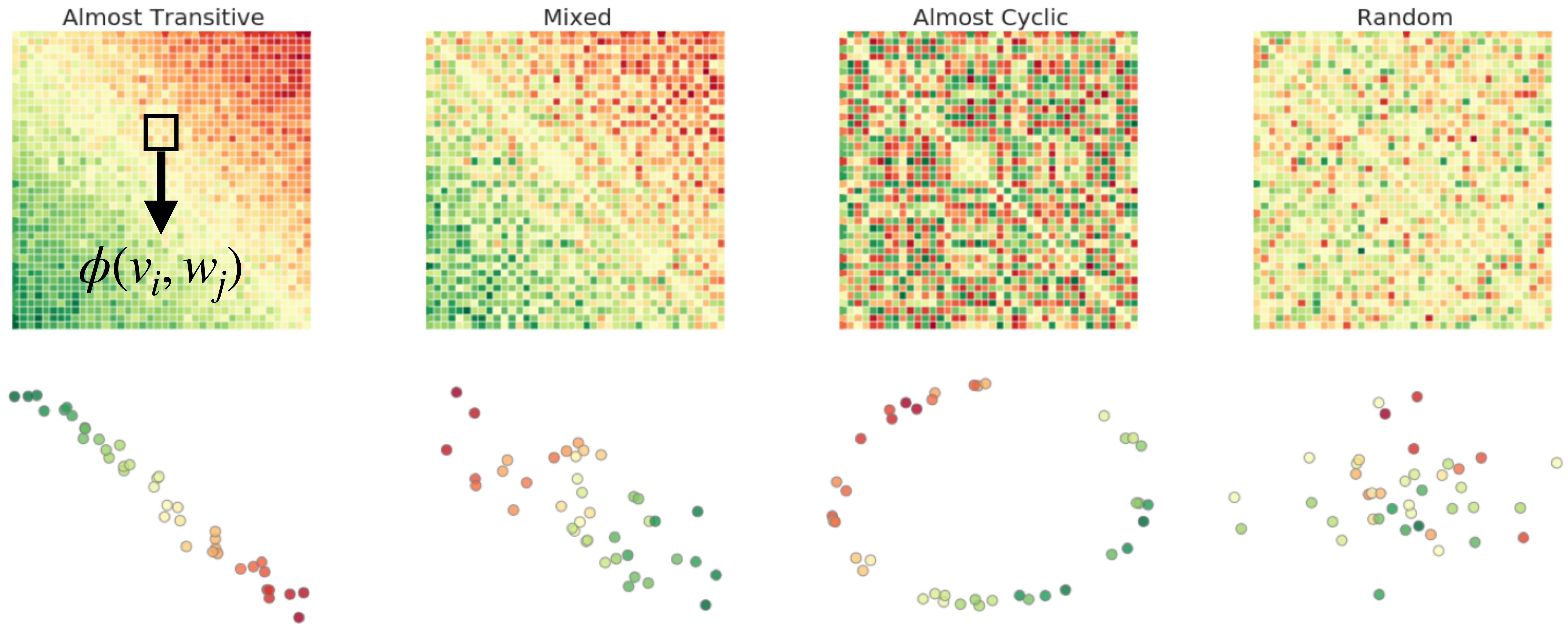
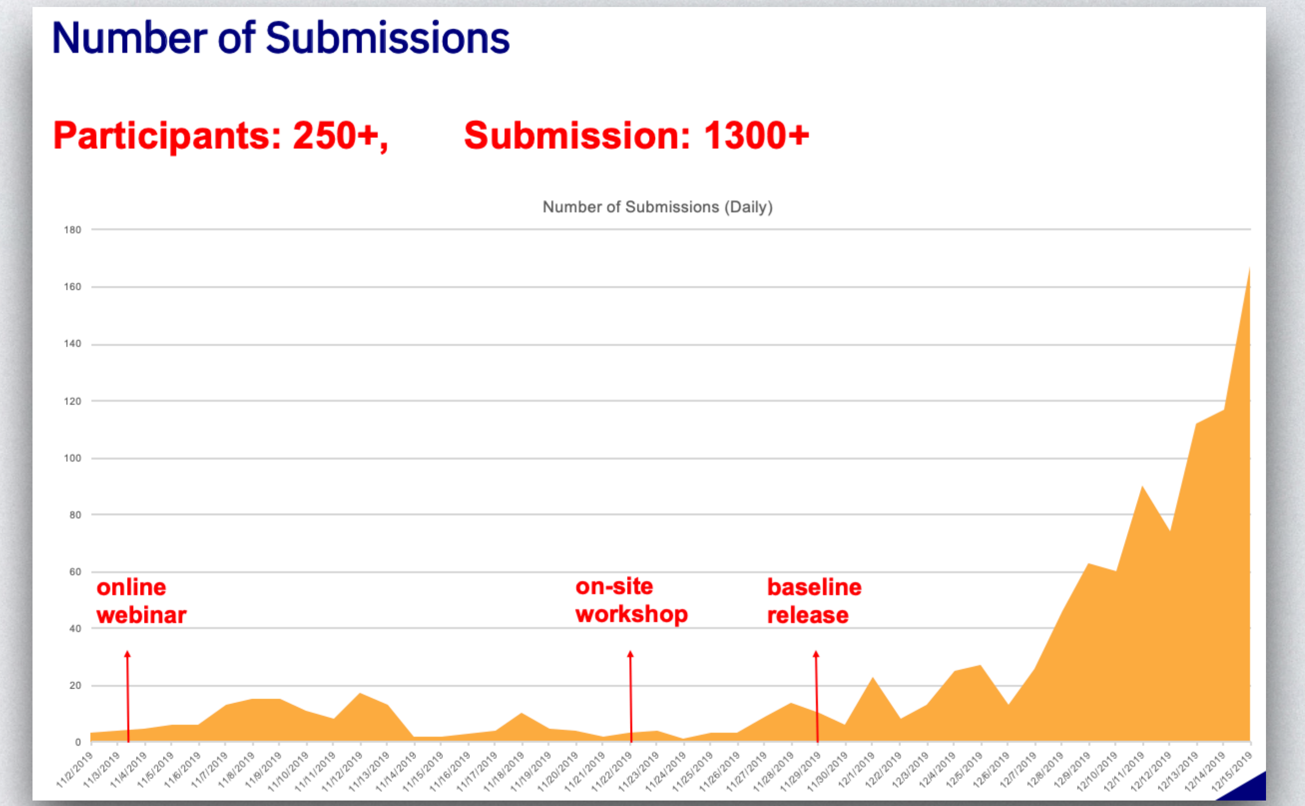


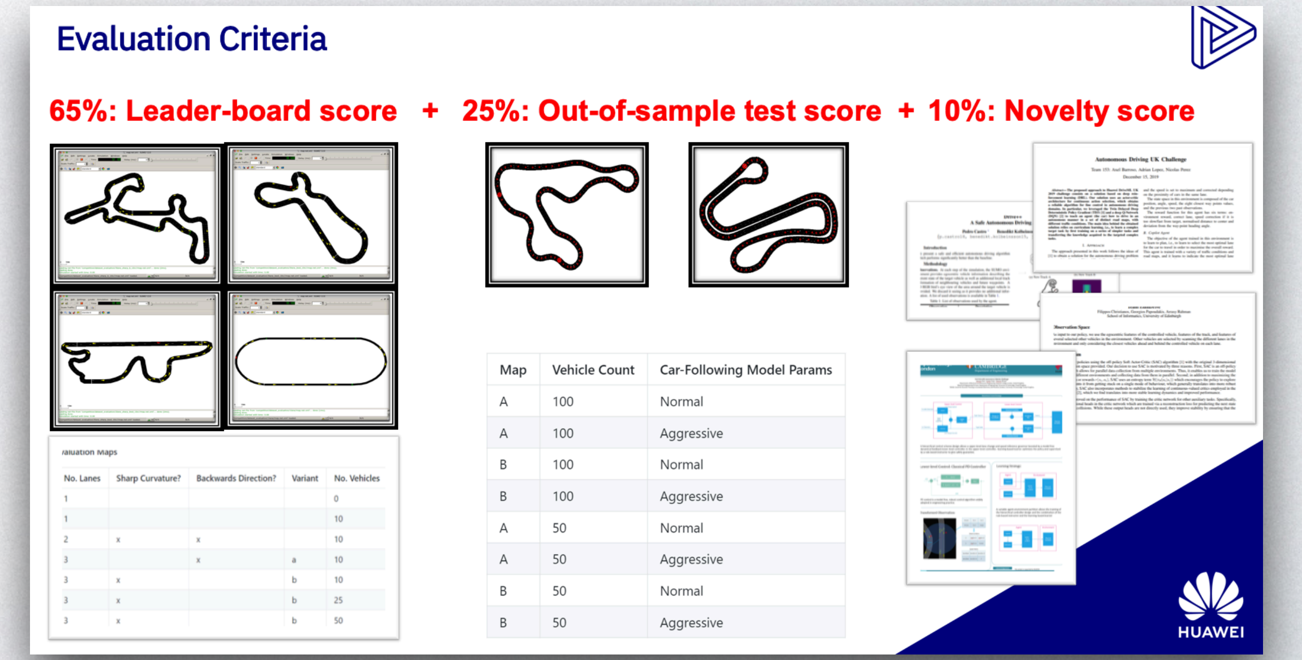
Figure 1. Low-dim gamescapes of various basic game structures. **Top row:** Evaluation matrices of populations of 40 agents each; colors vary from red to green as ϕ ranges over $[-1, 1]$. **Bottom row:** 2-dim embedding obtained by using first 2 dimensions of Schur decomposition of the payoff matrix; Color corresponds to average payoff of an agent against entire population; EGS of the transitive game is a line; EGS of the cyclic game is two-dim near-circular polytope given by convex hull of points. For extended version see Figure 6 in the Appendix.

[Balduzzi 2019]

Empirically, Self-play Did Not Work Either!

If we put the top-3 winner models together into one map, the top player will no longer perform the best.

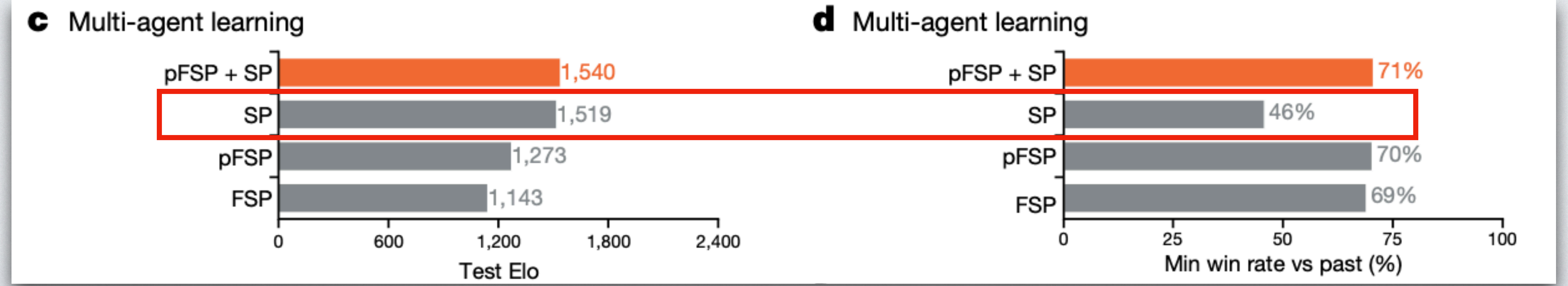




Empirically, Self-play Did Not Work Either!

Example on training AlphaStar:

- self-play can give you agents that are strong in terms of Elo, however, if one makes it compete against its previous strategies, it still loses.
- This shows that naive self-play will not work in real-world games simply because the cyclic dynamics, or, in other words, the agent will forget what has learned.



[Vinyals 2019, Table 3]

The Lesson: Understanding Game Structures are Critical !

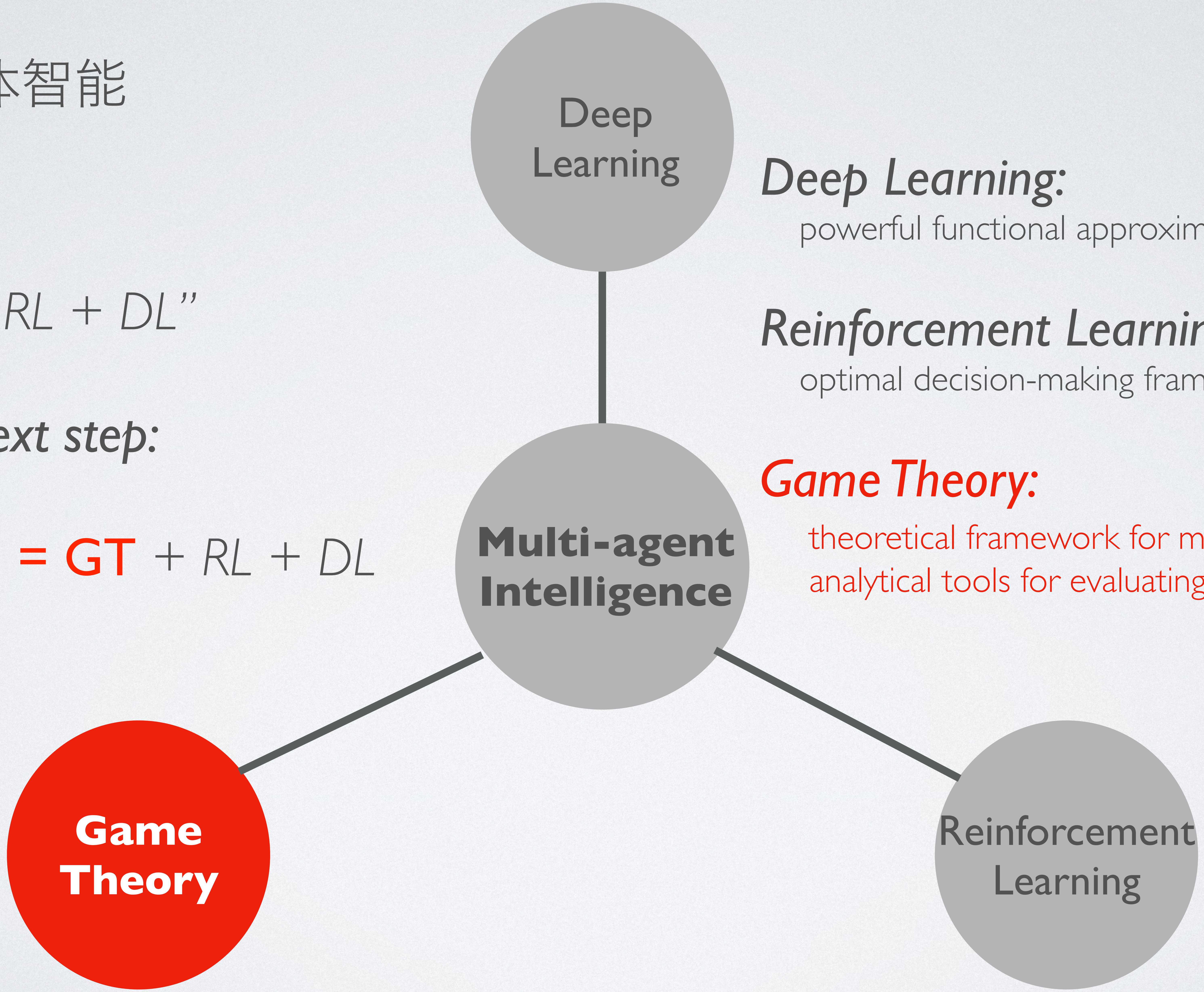
通用智能和群体智能

David Silver:

"AI = RL + DL"

I believe, in the next step:

Multi-agent AI = GT *+ RL + DL*



Deep Learning:
powerful functional approximator

Reinforcement Learning:
optimal decision-making framework

Game Theory:
theoretical framework for modelling multi-agent system
analytical tools for evaluating agents' policies

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  - ☐ Elo rating
  - ☐ Nash Equilibrium
  - ☐ Replicator dynamics
  - ☐ α-Rank & α^{α}-Rank

Recall Our Goal

Output: the reward $(R^1, \dots, R^N)$

Black-box multi-agent game engine

input

Our algorithm:

Multi-agent policy evaluation

Multi-agent policy improvement

output

"good" strategy $(\pi^{1,*} \dots, \pi^{N,*})$

Input: a joint strategy $(\pi^1, \dots, \pi^N)$

Real World Games Look Like Spinning Tops.

- Real-world games are mixtures of both transitive and in-transitive components, e.g., Go, DOTA, StarCraft II.
- Though winning is often harder than losing a game, finding a strategy that always loses is also challenging.
- Players who regularly practice start to beat less skilled players, this corresponds to the transitive dynamics.
- At certain level (the red part), players will start to find many different strategy styles. Despite not providing a universal advantage against all opponents, players will counter each other within the same transitive group. This provide direct information of improvement.
- As players get stronger to the highest level, seeing many strategy styles, the outcome relies mostly on skill and less on one particular game styles (以不变应万变).

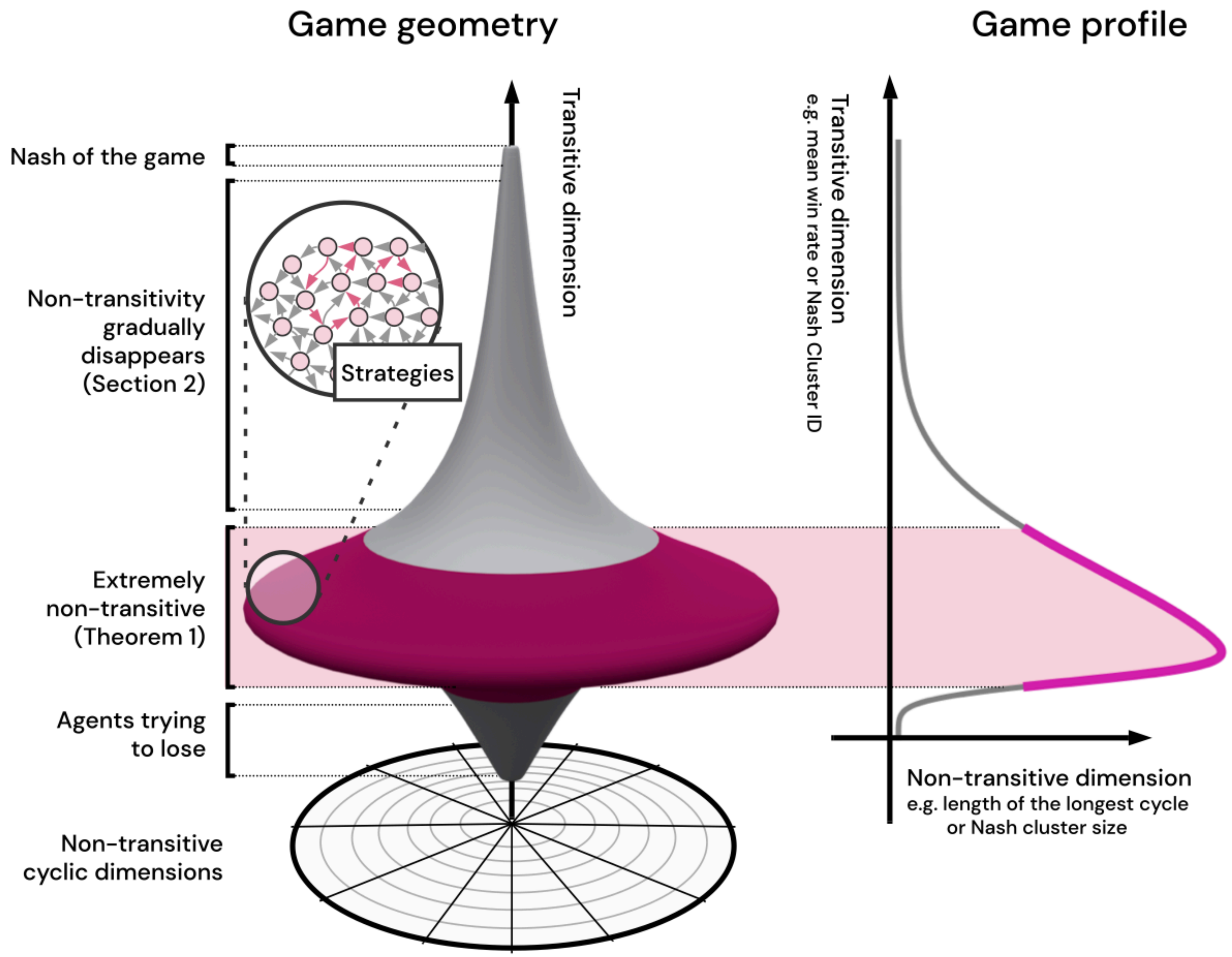
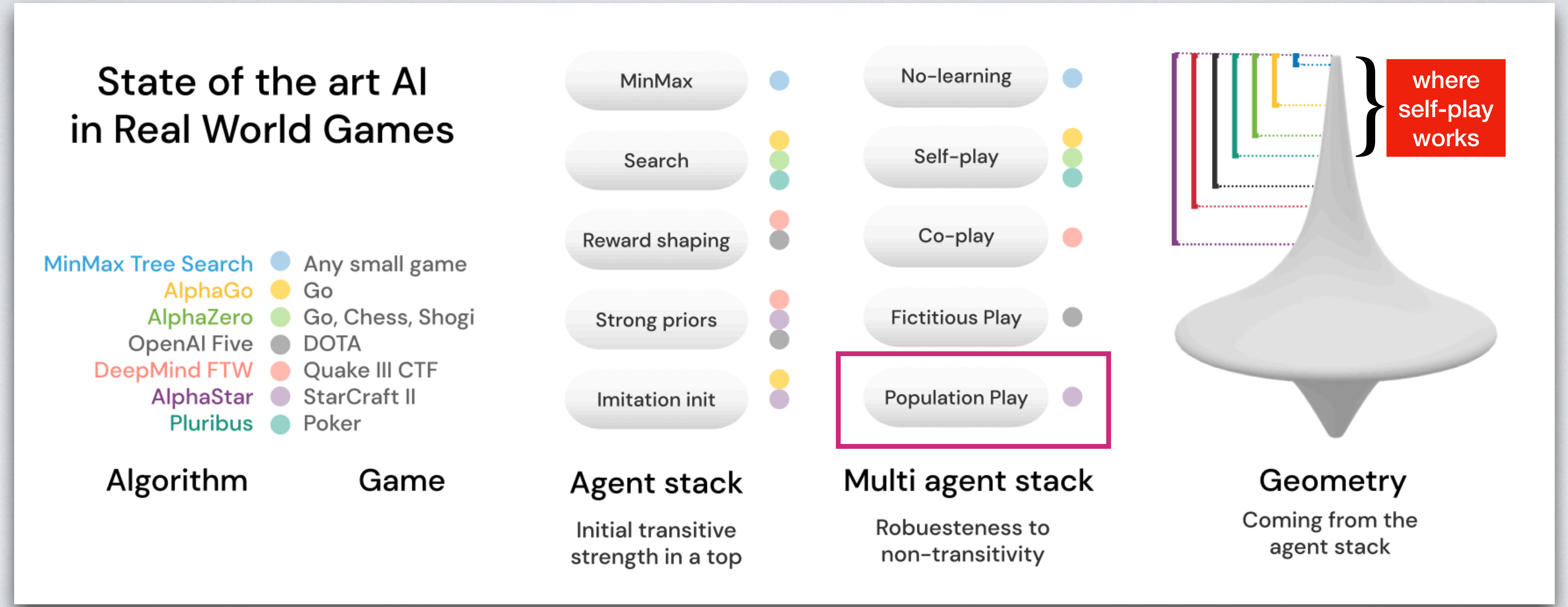


Figure 1: High-level visualisation of the geometry of Games of Skill. It shows a strong transitive dimension, that is accompanied by the highly cyclic dimensions, which gradually diminishes as skill grows towards the Nash Equilibrium (upward), and diminishes as skill evolves towards the worst possible strategies (downward). The simplest example of non-transitive behaviour is a cycle of length 3 that one finds e.g. in the Rock Paper Scissors game.

[Czarnecki 2020]

Understanding the game structure helps develop solutions

We should have a clear idea of why we use a method rather than hacking by trail and error from the beginning. Never use "reinforcement learning" to design reinforcement learning algorithms!



[Czarnecki 2020]

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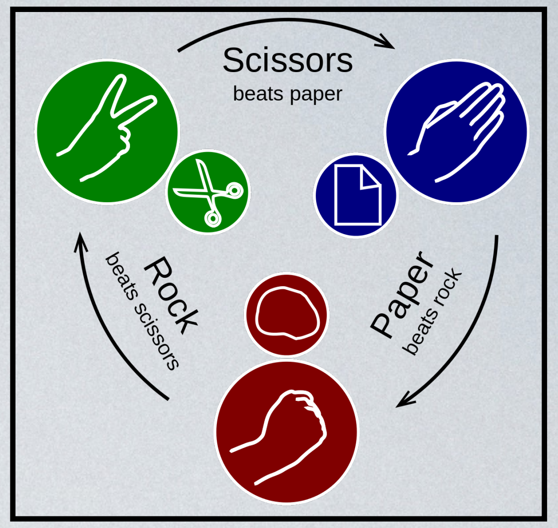
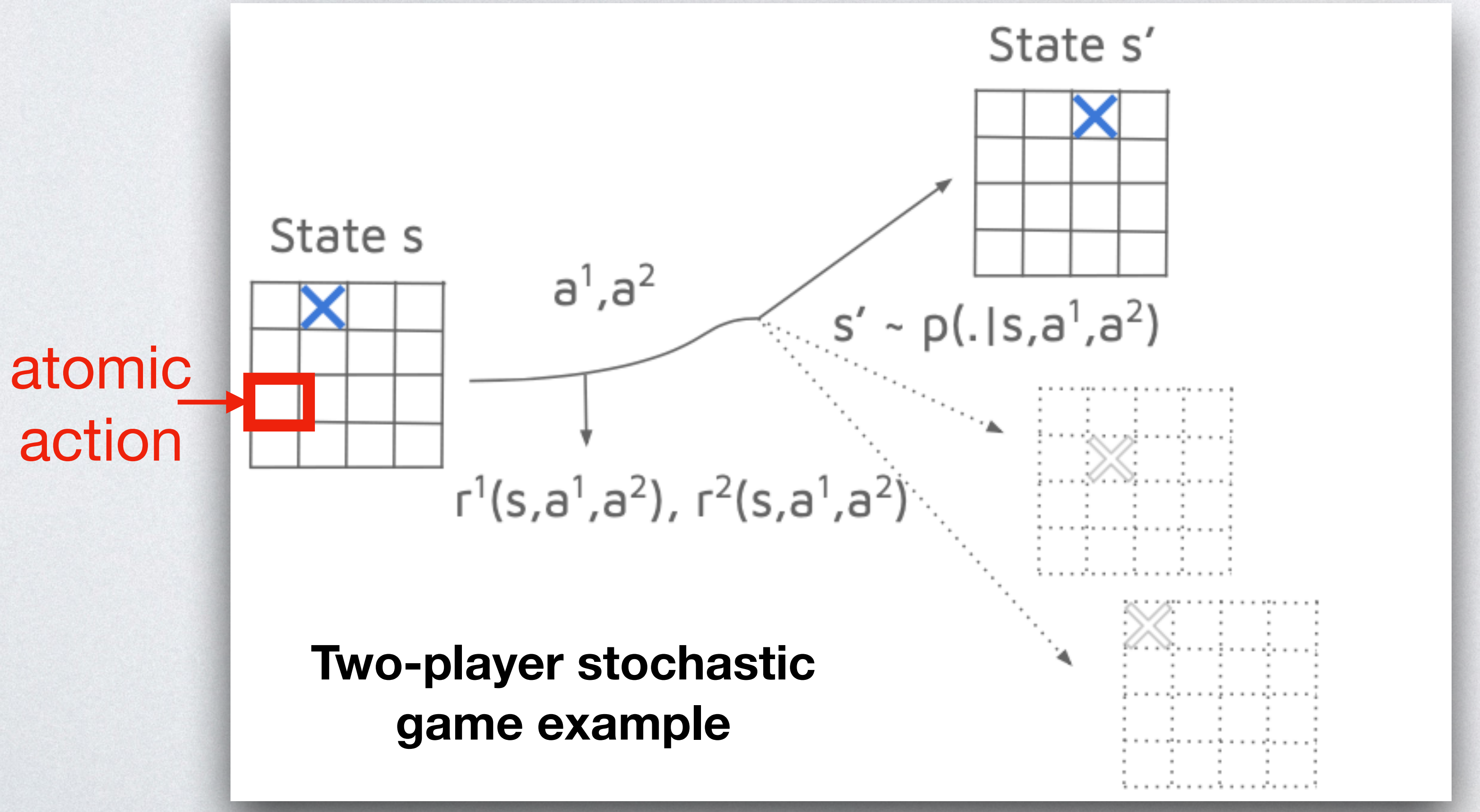
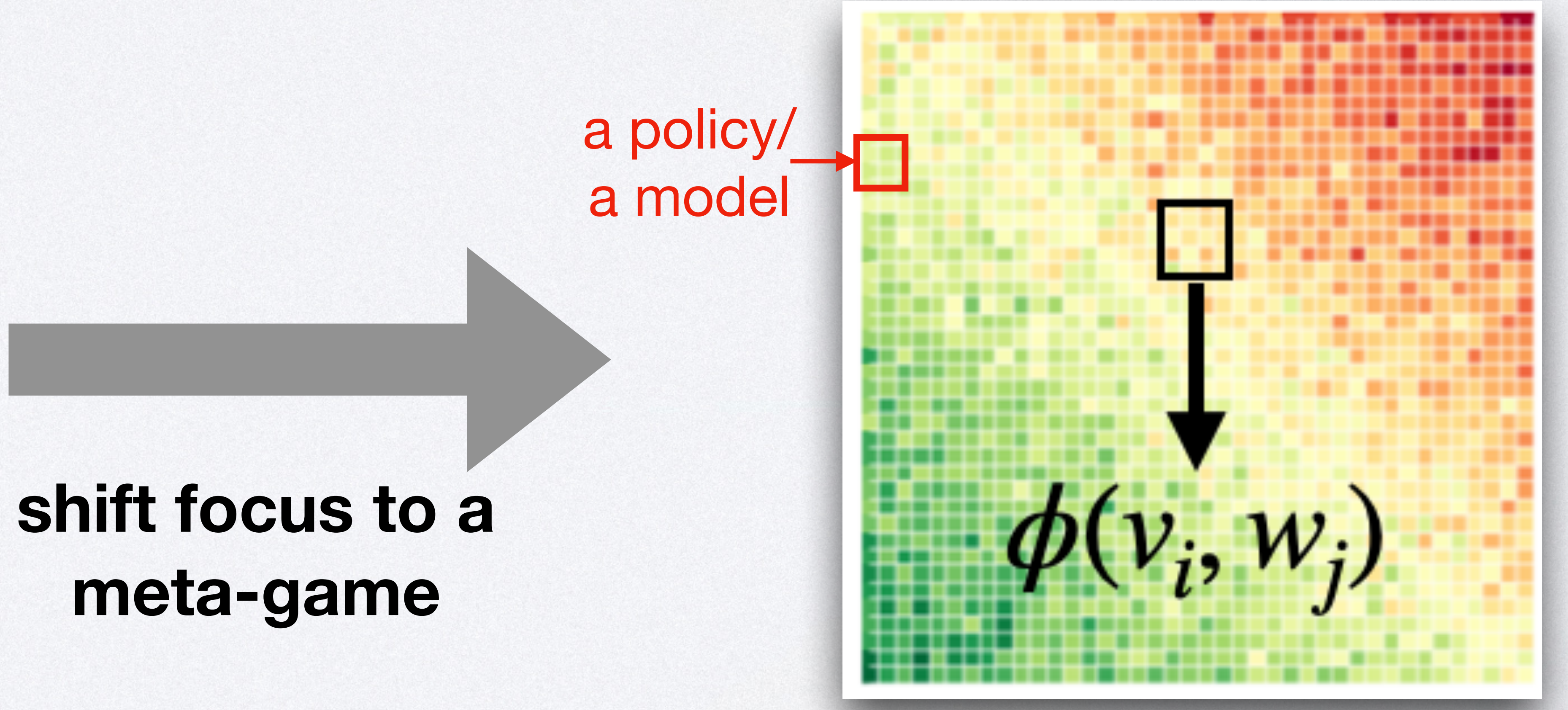
The Necessity of Studying Meta-games.



- An important intuition of solving games is to train many policies, a population of them. In RPS, if we have a population of three players, each of them plays R/P/S, and we randomise over which player to pick, then no one will ever be able to exploit us.
- On the other hand, enumerating every possible atomic state-action pairs is impossible for real-world games. We have to model on the higher-level policy level, e.g., aggressive/passive styles of policies, rather than state-action level.
- Understanding meta-games can help design both new games, and, new game solvers.
- It is called a **meta-game**, or, **empirical game**, or, **the problem problem**, or, **autocurricula**.





Terminology on Meta-Games.

- In the meta-game analysis, we assume a player can have many copies of itself, each of the copy can play different strategies.
- The "policy" in meta games mean how many copies of that player in the population play that particular type of policy, namely, a policy of policy.

Reinforcement Learning	Game Theory	Meta-game Analysis
environment	game	game
agent	player	population
action	action	type
policy	strategy	distribution over types
reward	payoff	fitness

How Does Meta-games Look Like

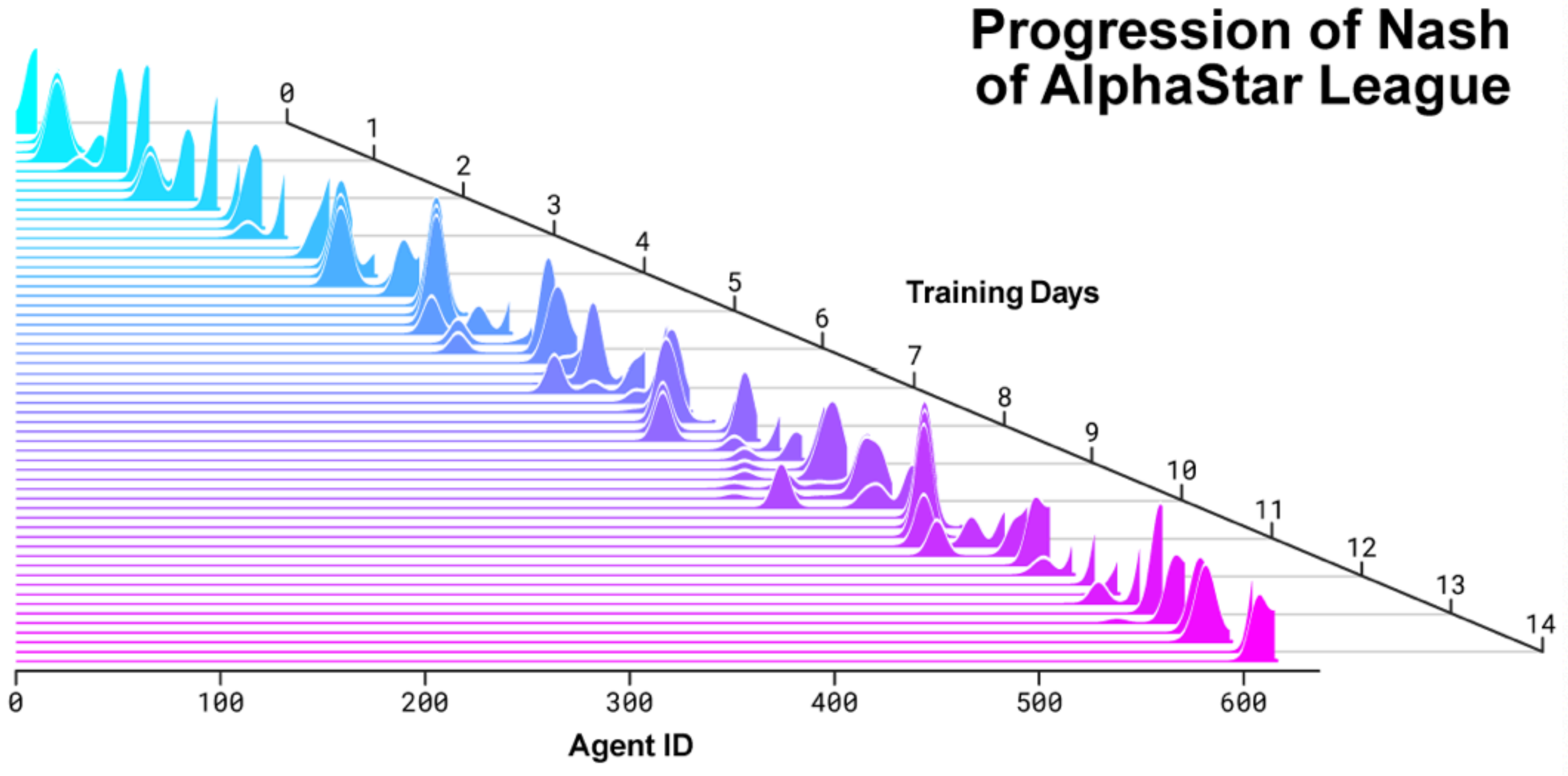
More examples of meta-games on AlphaGO and AlphaStar.

Extended Data Table 9 | Cross-table of win rates in per cent between programs

	α_{rvp}	α_{vp}	α_{rp}	α_{rv}	α_{r}	α_{v}	α_{p}
α_{rvp}	-	1 [0; 5]	5 [4; 7]	0 [0; 4]	0 [0; 8]	0 [0; 19]	0 [0; 19]
α_{vp}	99 [95; 100]	-	61 [52; 69]	35 [25; 48]	6 [1; 27]	0 [0; 22]	1 [0; 6]
α_{rp}	95 [93; 96]	39 [31; 48]	-	13 [7; 23]	0 [0; 9]	0 [0; 22]	4 [1; 21]
α_{rv}	100 [96; 100]	65 [52; 75]	87 [77; 93]	-	0 [0; 18]	29 [8; 64]	48 [33; 65]
α_{r}	100 [92; 100]	94 [73; 99]	100 [91; 100]	100 [82; 100]	-	78 [45; 94]	78 [71; 84]
α_{v}	100 [81; 100]	100 [78; 100]	100 [78; 100]	71 [36; 92]	22 [6; 55]	-	30 [16; 48]
α_{p}	100 [81; 100]	99 [94; 100]	96 [79; 99]	52 [35; 67]	22 [16; 29]	70 [52; 84]	-
CS	100 [97; 100]	74 [66; 81]	98 [94; 99]	80 [70; 87]	5 [3; 7]	36 [16; 61]	8 [5; 14]
ZN	99 [93; 100]	84 [67; 93]	98 [93; 99]	92 [67; 99]	6 [2; 19]	40 [12; 77]	100 [65; 100]

a policy/ a model → α_{rvp}

[Silver 2016, table 9]



THE NASH DISTRIBUTION OVER COMPETITORS AS THE ALPHASTAR LEAGUE PROGRESSED AND NEW COMPETITORS WERE CREATED. THE NASH DISTRIBUTION, WHICH IS THE LEAST EXPLOITABLE SET OF COMPLEMENTARY COMPETITORS, WEIGHTS THE NEWEST COMPETITORS MOST HIGHLY, DEMONSTRATING CONTINUAL PROGRESS AGAINST ALL PREVIOUS COMPETITORS.

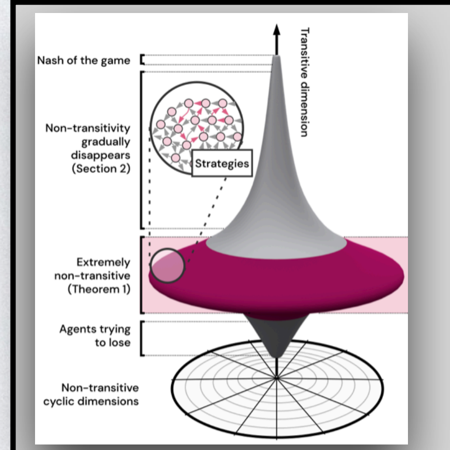
[AlphaStar blog]

The Target of Studying Meta-games.

- In the meta-game analysis, we can ask two critically important questions:

1. *How can we evaluate the population of policies in a meta-game, especially games with limited cycles?*

2. *How can we develop new policies based on the existing population of policies?*



Our algorithm:

1. Multi-agent policy evaluation

2. Multi-agent policy improvement

Relationships between Meta-games and Underlying games

- [Tuyls 2018] proved that a Nash for meta-game is an approximate Nash for the underlying game.
  - Define the Nash for the N-player K-strategy meta-game to be $\mathbf{x} = (x^1, \ldots, x^N)$, $\sum_{j=1}^{K} x_j^i = 1 \ \forall i \in N$.

$$E_{\pi \sim \mathbf{x}} \left[\hat{r}^i(\pi) \right] = \max_{\pi^i} E_{\pi^{-i} \sim x^{-i}} \left[\hat{r}^i \left(\pi^i, \pi^{-i} \right) \right], \forall i \in N$$

  - If we define the reward of the underlying game to be $r^i(\pi^i, \pi^{-i})$, $r^i = \mathbf{E}[\hat{r}^i]$, and $\epsilon = \sup_{\boldsymbol{\pi}, i} | \hat{r}^i(\boldsymbol{\pi}) - r^i(\boldsymbol{\pi}) |$

Distance to the Nash of the underlying game

$$\max_{\pi} E_{\pi^{-i} \sim x^{-i}} \left[r^i \left(\pi^i, \pi^{-i} \right) \right] - E_{\pi \sim x} \left[r^i(\pi) \right]$$

$$\leq \underbrace{\max_{\pi^i} E_{\pi^{-i} \sim x^{-i}} \left[\hat{r}^i \left(\pi^i, \pi^{-i} \right) \right] - E_{\pi \sim x} \left[\hat{r}^i(\pi) \right]}_{=0 \text{ since } x \text{ is a Nash equilibrium for } \hat{r}^i} + \underbrace{\max_{\pi^i} E_{\pi^{-i} \sim x^{-i}} \left[r^i \left(\pi^i, \pi^{-i} \right) - \hat{r}^i \left(\pi^i, \pi^{-i} \right) \right]}_{\leq \epsilon} \underbrace{- E_{\pi \sim x} \left[r^i(\pi) - \hat{r}^i(\pi) \right]}_{\leq \epsilon}$$

$$\leq 2\epsilon$$

- One can further use Hoeffding equation to have a finite-sample bound on how many samples n are needed in order to control ϵ with high probability $1 - \delta$.

$$P\left(\sup_{\boldsymbol{\pi}, i} \left| r^i(\boldsymbol{\pi}) - \hat{r}^i(\boldsymbol{\pi}) \right| < \epsilon \right) \geq \left(1 - 2e^{\left(-2\epsilon^2 n\right)} \right)^{K^{N+1}}$$

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Policy Evaluation on Meta Games via Elo Ratings

- Elo create a rating $(r_1, \ldots r_N)$ by averaging the historical performance. Assuming the true probability of agent i beating agent j is p_{ij}, Elo approximates it by $\hat{p}_{ij} = \text{softmax}(r_i - r_j)$ through minimising the cross entropy by

$$\ell_{\text{Elo}}\left(p_{ij}, \hat{p}_{ij}\right) = -p_{ij} \log \hat{p}_{ij} - \left(1 - p_{ij}\right) \log\left(1 - \hat{p}_{ij}\right)$$

- Suppose the t-th match pits i against j, and binary outcome is $S^t_{i,j}$, then the rating updates

$$r_i^{t+1} \leftarrow r_i^t - \eta \cdot \nabla_{r_i} \ell_{\text{Elo}}\left(S^t_{ij}, \hat{p}^t_{ij}\right) = r_i^t + \eta \cdot \left(S^t_{ij} - \hat{p}^t_{ij}\right)$$

- With enough race data, Elo ratings will converge to $p_{ij} = \bar{p}_{ij} = \sum_n \frac{S^n_{ij}}{N_{ij}}$, historical average.
- Elo cannot deal with in-transitive games, since $\text{curl}(\text{logit}\mathbf{P}) = 0$.
- In RPS, p_{ij} is (1/2, 1/2, 1/2), thus no predictive power about the game.
- Elo can be biased by weak players that intend to lose (刷分水军/演员) [Balduzzi 2018].

Policy Evaluation on Meta Games via Nash Equilibrium

- Treat meta game as a normal-form game, and compute Nash equilibrium by LP.
- In two-player zero-sum discrete case, it can be solved in polynomial time. The matrix $\mathbf{A}_{\mathfrak{P}}$ is anti-symmetrical, i.e., $\mathbf{A}_{\mathfrak{P}} = -\mathbf{A}_{\mathfrak{P}}^{\top}$.

$$\mathbf{A}_{\mathfrak{P}} := \left\{ \phi\left(\mathbf{w}_i, \mathbf{w}_j\right) : \left(\mathbf{w}_i, \mathbf{w}_j\right) \in \mathfrak{P} \times \mathfrak{P} \right\} =: \phi(\mathfrak{P} \otimes \mathfrak{P})$$

- The minimax theorem is a natural outcome of the duality theorem in LP.

Prime problem

$$\max_{v \in \mathbb{R}} v$$
$$\text{s.t. } \mathbf{p}^{\top}\mathbf{A}_{\mathfrak{P}} \geq v \cdot \mathbf{1}$$
$$\mathbf{p} \geq \mathbf{0} \text{ and } \mathbf{p}^{\top}\mathbf{1} = 1$$

/

Dual problem

$$\min_{v \in \mathbb{R}} v$$
$$\text{s.t. } \mathbf{q}^{\top}\mathbf{A}_{\mathfrak{P}}^{\top} \leq v \cdot \mathbf{1}$$
$$\mathbf{q} \geq \mathbf{0} \text{ and } \mathbf{q}^{\top}\mathbf{1} = 1$$

⟺

Minimax theorem

$$\max_{\mathbf{p}} \min_{\mathbf{q}} \mathbf{p}^{\top}\mathbf{A}_{\mathfrak{P}}\mathbf{q} = \min_{\mathbf{q}} \max_{\mathbf{p}} \mathbf{p}^{\top}\mathbf{A}_{\mathfrak{P}}\mathbf{q}$$

Policy Evaluation on Meta Games via Nash Equilibrium

- Cons of Nash equilibrium:
  - Only tractable in two-player zero-sum tabular case. Multi-player general-sum is PPAD-hard.
  - It is a fixed point due to the Brouwer fix-point theorem.
  - What Nash can tell, including its generalisation such as correlated or coarse correlate equilibrium, is the time-averaged behaviour; it tells us little about the "dynamical" behaviour of the actual system.
  - But some dynamics will not only converge to Nash, but they also cycle. Or, they do not end up with Nash at all. The following theorem can summarise.

Poincaré–Bendixson Theorem:

Given a differentiable real dynamical system defined on an open subset of the plane, every non-empty compact ω-limit set of an orbit, which contains only finitely many fixed points, is either

- ***a fixed point***
- ***a periodic orbit***
- ***a connected set composed of a finite number of fixed points together with homoclinic and heteroclinic orbits connecting these.***

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Policy Evaluation on Meta Games via Replicator Dynamics

- Replicator dynamics is a framework of dynamical system that describes the time dependencies of the players' behaviours.
  - Think of an infinitely-sized population of agents, let x_k be the proportion of agents in the population who play the k^{th} strategy among $K-$many possible strategies. In a two-player (i.e. two populations) game, let $(\mathbf{A}, \mathbf{B})$ be the payoff matrix, RD describes the continuous-time evolution of (x_k, y_k).
  - RD only works in symmetrical game $\mathbf{A} = \mathbf{B}^\top$ or anti-symmetrical game $\mathbf{A} = -\mathbf{B}^\top$.

payoff for the k^{th} strategy

$$\frac{dx_k}{dt} = x_k \left[(\mathbf{Ay})_k - \mathbf{x}^T\mathbf{Ay}\right], \quad \frac{dy_k}{dt} = y_k \left[\left(\mathbf{x}^T\mathbf{B}\right)_k - \mathbf{x}^T\mathbf{By}\right]$$

current proportion, replicating itself

current payoff against the opponent population

payoff matrix for the other population

Physical Meaning of Replicator Dynamics

- Replicator dynamics is deeply rooted with reinforcement learning.
  - In Cross Learning and finite action-set automata (RL back to the old times), with normalised reward, $0 \leq r \leq 1$, we have the learning rule of the probability of selecting the i-th action as:

$$\pi(i) \leftarrow \pi(i) + \begin{cases} r - \pi(i)r & \text{if } i = j \\ -\pi(i)r & \text{otherwise} \end{cases}$$

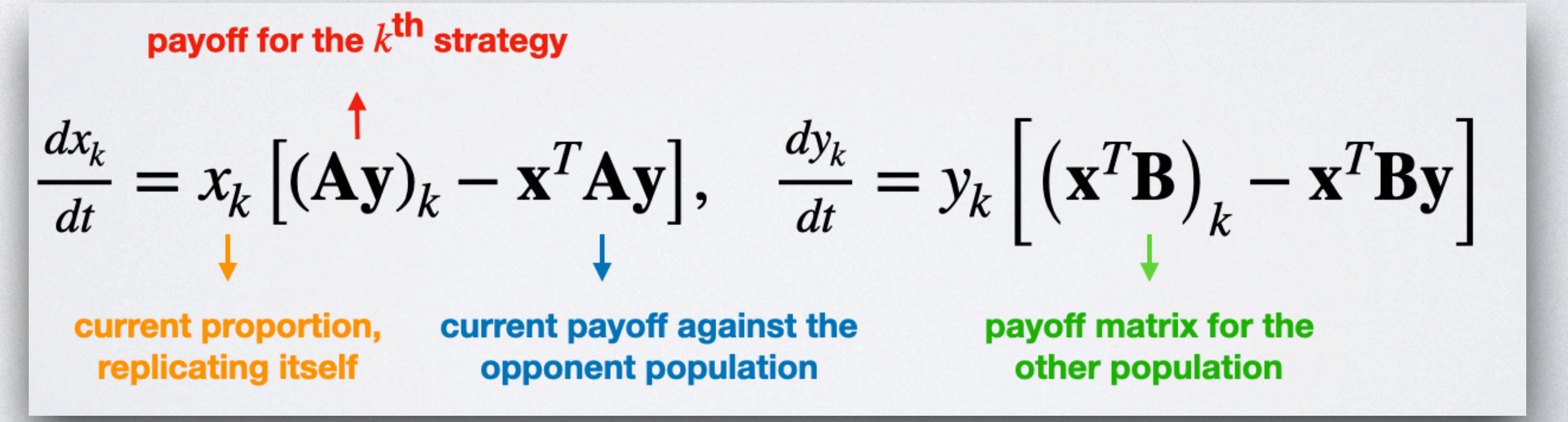
  - We can then write the expected change in policy i by:

$$E[\Delta\pi(i)] = \pi(i)\left[E_i[r] - \pi(i)E_i[r]\right] + \sum_{j \neq i} \pi(j)\left[-E_j[r]\pi(i)\right]$$

$$= \pi(i)\left[E_i[r] - \sum_j \pi(j)E_j[r]\right]$$

  - Assuming to take infinitesimal step $\lim \delta \rightarrow 0$ in $\pi_{t+\delta}(i) = \pi_t(i) + \delta\Delta\pi_t(i)$, we have

$$\dot{\pi}(i) = \pi(i)\left[\mathrm{E}_i[r] - \sum_j \pi(j)\mathrm{E}_j[r]\right]$$



Physical Meaning of Replicator Dynamics

- Replicator dynamics is deep rooted with reinforcement learning.
  - Q-learning can be derived equivalently as a variant of RD with exploration [Kianercy 2012].
  - In the stateless RL setting, one can write Q-learning update rule as

$$Q_i(t+1) = Q_i(t) + \alpha \left[r_i(t) - Q_i(t) \right]$$

Note, no $\max$ is needed here!

  - the continuous limit of the above update rule is

$$\dot{Q}_i(t) = \alpha \left[r_i(t) - Q_i(t) \right]$$

  - and naturally, the policy withe exploration is written as

$$x_i(t) = \frac{e^{Q_i(t)/T}}{\sum_k e^{Q_k(t)/T}}, i = 1,2,\cdots,n$$

  - differentiating the Boltzmann policy w.r.t to time, we can have

$$\frac{\dot{x}_i}{x_i} = [r_i - \sum_{k=1}^n x_k r_k] - T \sum_{k=1}^n x_k \ln \frac{x_i}{x_k}$$

  - plug in the reward functions

$$\dot{x}_i = x_i\left[(A\mathbf{y})_i - \mathbf{x}\cdot A\mathbf{y} + T_X \sum_j x_j \ln(x_j/x_i)\right]$$

$$\dot{y}_i = y_i\left[(B\mathbf{x})_i - \mathbf{y}\cdot B\mathbf{x} + T_Y \sum_j y_j \ln(y_j/y_i)\right]$$

New term on entropy

payoff for the k^{th} strategy

$$\frac{dx_k}{dt} = x_k \left[(\mathbf{Ay})_k - \mathbf{x}^T\mathbf{Ay} \right], \quad \frac{dy_k}{dt} = y_k \left[(\mathbf{x}^T\mathbf{B})_k - \mathbf{x}^T\mathbf{By} \right]$$

current proportion, replicating itself

current payoff against the opponent population

payoff matrix for the other population

“*Perhaps a thing is simple if you can describe it fully in several different ways, without immediately knowing that you are describing the same thing*” —— R. Feynman

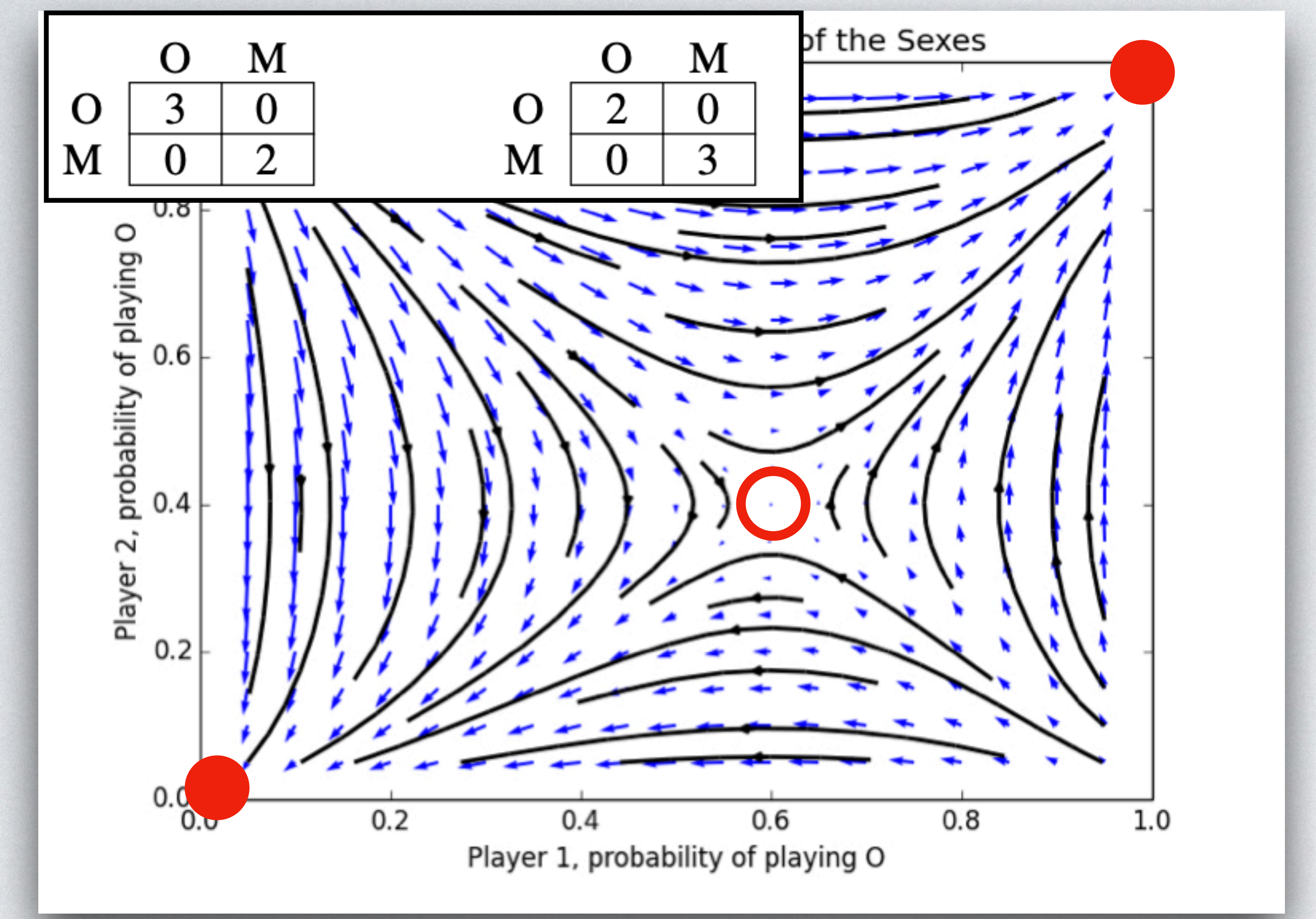
- Many RL algorithms are equivalent to the variants of replicator dynamics.
  - Besides Q-learning, policy gradient can also be written as RD [Hennes 2020].

Table 4: An overview of related empirical evaluations of learning dynamics. NFG: normal-form games; CNFG: continuous action normal-form games; SG: stochastic (Markov) games.

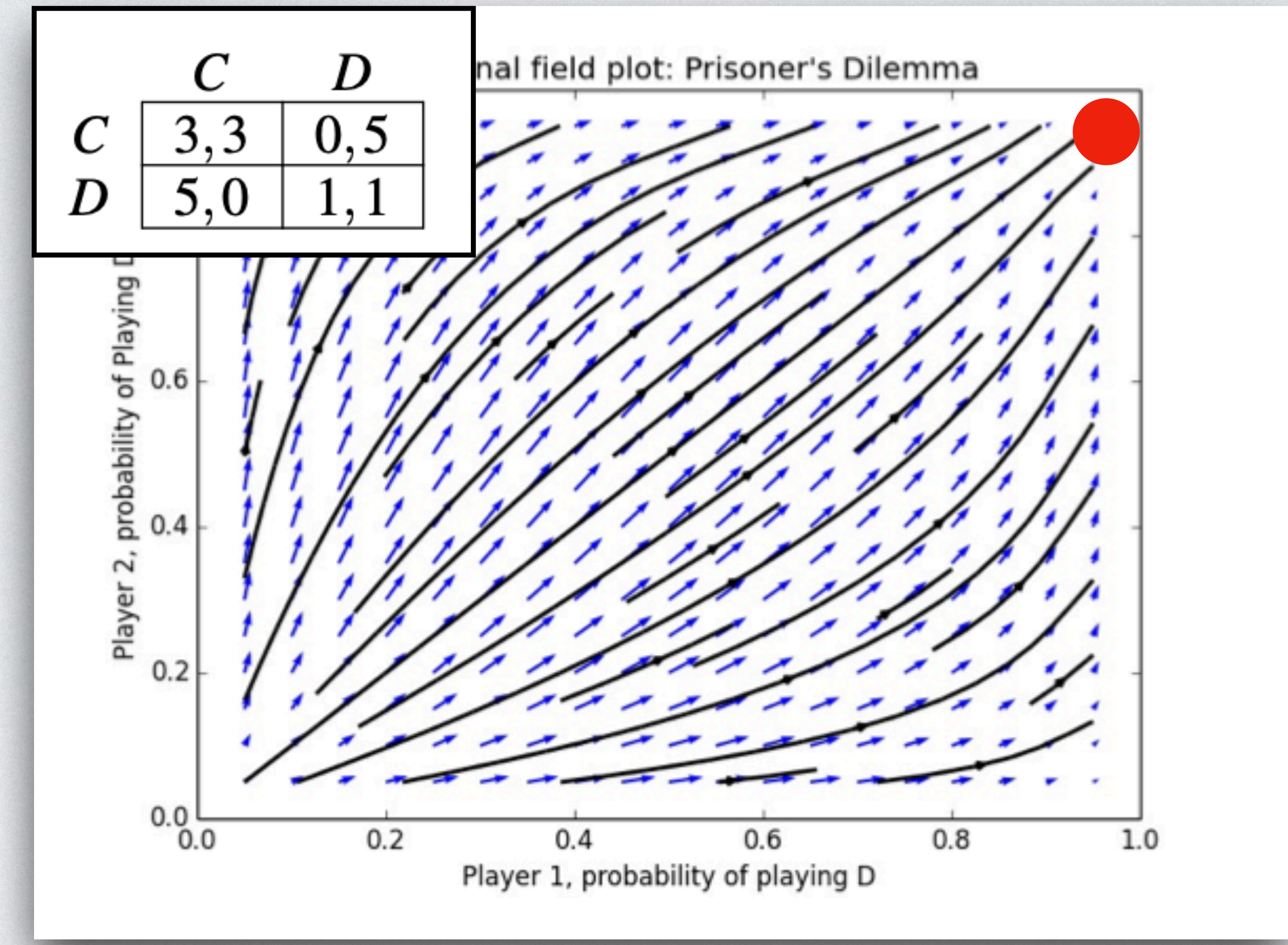
Type	Algorithm	Reference
NFG	Q-learning	Tuyls et al. (2003, 2006)
NFG	regret minimisation	Klos et al. (2010)
NFG	FAQ	Kaisers and Tuyls (2010, 2011)
NFG	lenient FAQ	Bloembergen et al. (2011) Kaisers (2012)
NFG	WoLF	Bowling and Veloso (2002)
NFG	IGA, IGA-WoLF, WPL	Abdallah and Lesser (2008)
CNFG	Q-learning	Galstyan (2013)
SG	networks of learning automata	Vrancx et al. (2008a) Hennes et al. (2009)
SG	RESQ-learning	Hennes et al. (2010)

[Bloembergen 2015]

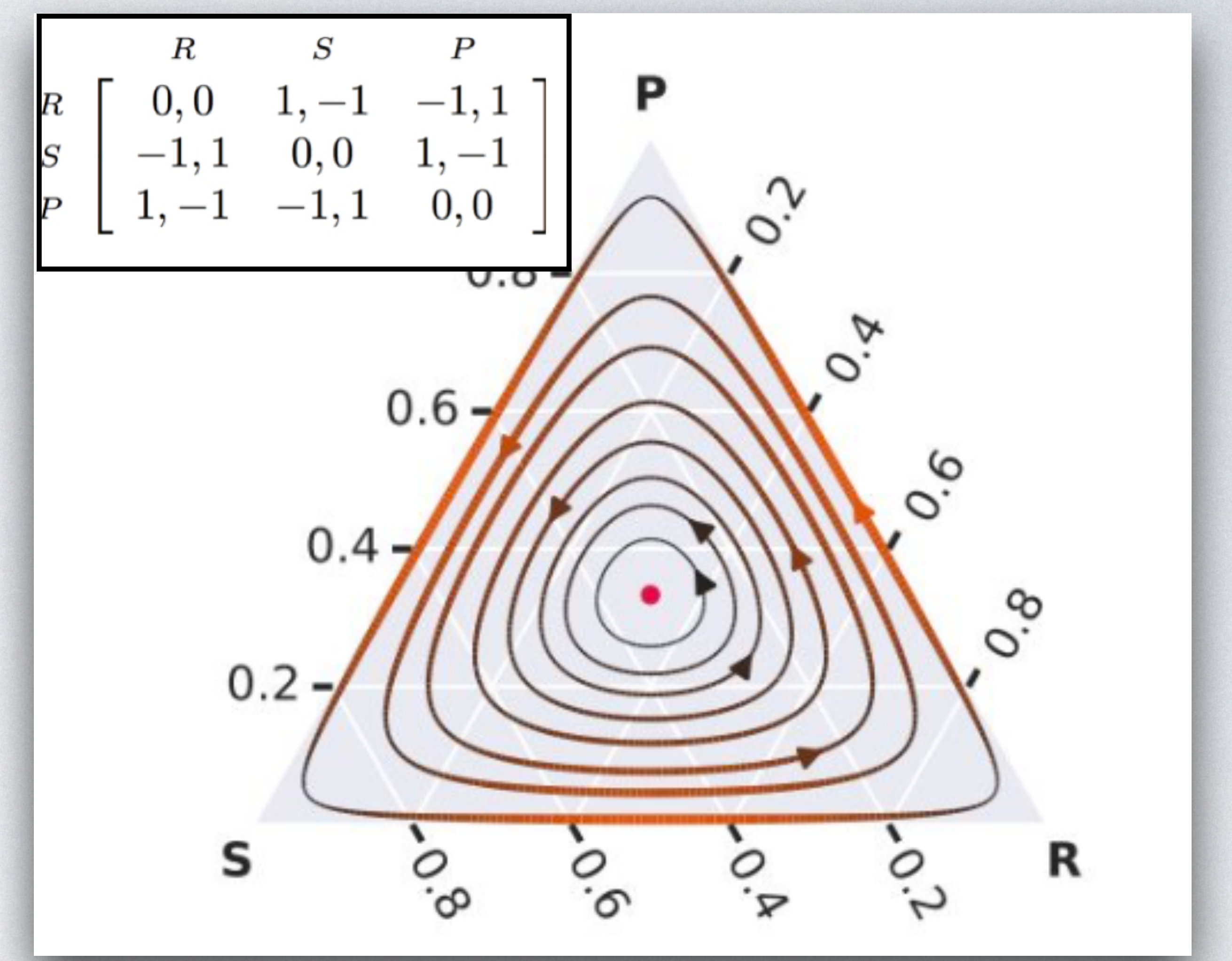
What does Replicator Dynamics suggest



Battle of sexes



Prison's Dilemma



Rock-Paper-Scissor

Extended Data Table 9 | Cross-table of win rates in per cent between programs

	α_{rvp}	α_{vp}	α_{rp}	α_{rv}	α_r	α_v	α_p
α_{rvp}	-	1 [0; 5]	5 [4; 7]	0 [0; 4]	0 [0; 8]	0 [0; 19]	0 [0; 19]
α_{vp}	99 [95; 100]	-	61 [52; 69]	35 [25; 48]	6 [1; 27]	0 [0; 22]	1 [0; 6]
α_{rp}	95 [93; 96]	39 [31; 48]	-	13 [7; 23]	0 [0; 9]	0 [0; 22]	4 [1; 21]
α_{rv}	100 [96; 100]	65 [52; 75]	87 [77; 93]	-	0 [0; 18]	29 [8; 64]	48 [33; 65]
α_r	100 [92; 100]	94 [73; 99]	100 [91; 100]	100 [82; 100]	-	78 [45; 94]	78 [71; 84]
α_v	100 [81; 100]	100 [78; 100]	100 [78; 100]	71 [36; 92]	22 [6; 55]	-	30 [16; 48]
α_p	100 [81; 100]	99 [94; 100]	96 [79; 99]	52 [35; 67]	22 [16; 29]	70 [52; 84]	-
CS	100 [97; 100]	74 [66; 81]	98 [94; 99]	80 [70; 87]	5 [3; 7]	36 [16; 61]	8 [5; 14]
ZN	99 [93; 100]	84 [67; 93]	98 [93; 99]	92 [67; 99]	6 [2; 19]	40 [12; 77]	100 [65; 100]

AlphaGo meta game

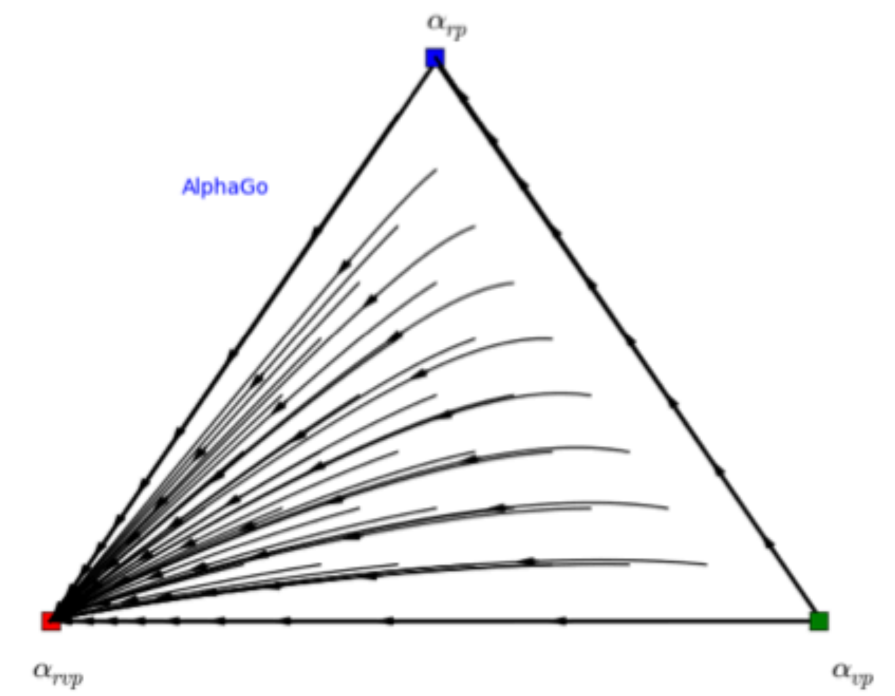
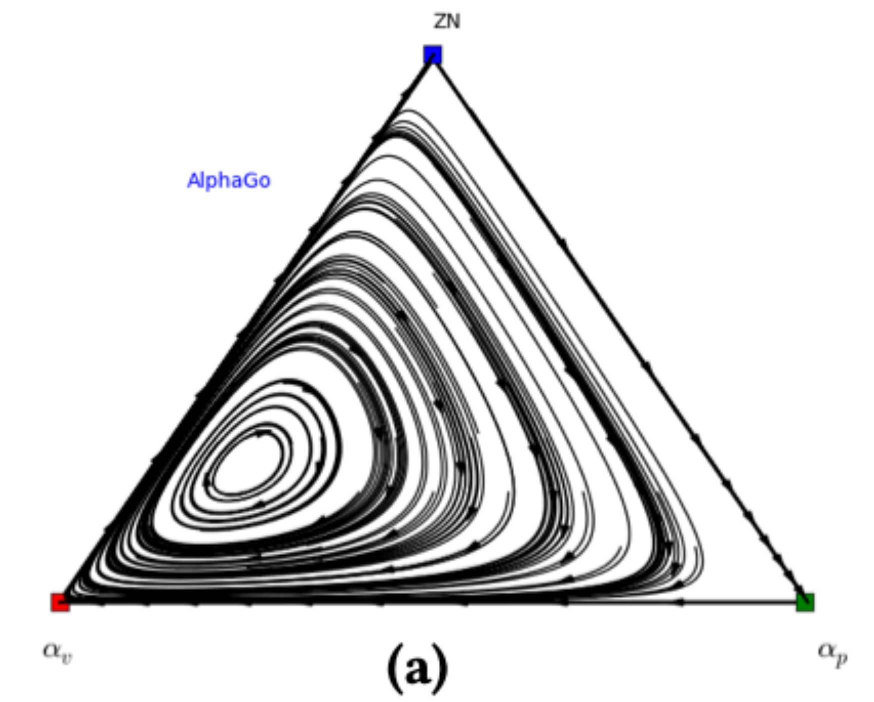


Figure 2: Trajectory plot for the 2-face consisting of strategies α_{rvp}, α_{vp}, α_{rp}

AlphaGo version comparison



(a)

Figure 5: Intransitive behaviour for α_v, α_p, and Zen.

AlphaGo version comparison

[Tuyls 2018]

Solution Concept of Replicator Dynamics

- The equilibrium points of replicator dynamics is evolutionary stable strategy (ESS).
  - ESS is new way to define "optimality", similar to the optimality defined in Nash means best response.
  - ESS means the strategy cannot be invaded by any alternative strategies from natural selection.
  - ESS is a refinement of Nash, it is a special type of Nash that is evolutionary stable.
  - On a symmetrical game, Nash equilibrium is:

$$R(\pi, \pi) \geq R(\pi', \pi), \quad \pi' \neq \pi$$

  - ESS refines Nash: $R(\pi, \pi) \geq R(\pi', \pi) \ \& \ R(\pi, \pi') \geq R(\pi', \pi'), \quad \pi' \neq \pi$
  - Examples of Nash that is not ESS, (A,A)/(B,B) are Nash but only (B,B) is ESS. *A* is not an ESS, so *B* can neutrally invade a population of *A* strategists and predominate, because *B* scores higher against *B* than *A* does against *B*.

	A	B
A	2, 2	1, 2
B	2, 1	2, 2

Harm thy neighbor

A cannot dominate B, since R(B,A)=R(A,A)
but B can dominate A, since R(B,B)>R(A,B)

Pros & Cons of Replicator Dynamics

- Pros of RD
  - RD offers continuous-time dynamics, compared to fixed point Nash, provide insights into micro-dynamical structures of games, e.g., flows, basins of attraction, and equilibria.
  - It provides a new angel to evaluate the policies in a game from a population perspective.
  - The solution concept describes the stability in the sense of evolution (优胜劣汰).
  - It can sift out unstable Nash equilibrium, e.g. the (2/5, 3/5) in battle of sexes.

- Cons of RD
  - It can only apply on two-player several-policy meta game due to the inherently-coupled dynamics.
  - It cannot work on general-sum games, the payoff has to be either symmetrical game $A = B^\top$, or asymmetrical games $A = -B^\top$.
  - The equilibrium is not unique.

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Weakness of Evaluation Metrics for Meta-games so far.

- **Elo rating:**
  - cannot deal with in-transitive games.
  - cannot tell the dynamics of strategy strength/weakness.
  - cannot stay unbiased to redundant weak agents.

- **Nash equilibrium:**
  - cannot scale to more than two players in non-zerosum games.
  - cannot guarantee uniqueness of equilibrium.
  - cannot tell the dynamics of strategy strength/weakness.

- **Replicator dynamics:**
  - cannot scale to more than two players.
  - cannot deal with general-sum games (either $A = B^\top$ or $A = -B^\top$).
  - cannot guarantee uniqueness of equilibrium.

- **Key requirements:** in-transitive, dynamical, multi-player, general-sum, tractable, unique, stable.

α-Rank: A General Solution Concept for Game Evaluation

α-Rank: Multi-Agent Evaluation by Evolution

father of PPAD class

Shayegan Omidshafiei*[1], Christos Papadimitriou*[3], Georgios Piliouras*[2], Karl Tuyls*[1], Mark Rowland[1], Jean-Baptiste Lespiau[1], Wojciech M. Czarnecki[1], Marc Lanctot[1], Julien Perolat[1], and Remi Munos[1]

[1] DeepMind, 6 Pancras Square, London, UK
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[3] Columbia University, New York, USA
* Equal contributors, ordered alphabetically. Corresponding author: Karl Tuyls <karltuyls@google.com>.

α-Rank is a new type of evaluation metric that can

- deal with both transitive and in-transitive game dynamics.
- model the flow of dynamics of strategy evolutions, rather than being a fixed point.
- scale to multi-player general-sum cases.
- tractable to be computed, equilibrium can be solved in polynomial time w.r.t the size of meta game.
- equilibrium point is unique, and, (evolutionary) stable.

α-Rank: A General Solution Concept for Game Evaluation

- We knew functional-form games and normal-form games can be decomposed:

[Balduzzi 2019] FFG = Transitive game $\oplus$ In-transitive/Cyclic game

[Candogan 2010] Normal-form Game = Potential Game $\oplus$ Hamonic Game

Unifying them can be a very good research topic 😄

- α-Rank is inspired by the Conley's fundamental theorem on dynamical system:

[Conley1978] Any flow on a compact metric space decomposes into a gradient-like part that leads to a recurrent part

- This suggests that a flow is either a part of a "*recurrent chain*", or on its way to converge to a "*recurrent chain*".

- The "recurrent *chain" component* of a game corresponds to the Sink Strongly Connected Component (SSCC) of the response graph.

The Sink Strongly Connected Component of the Response Graph

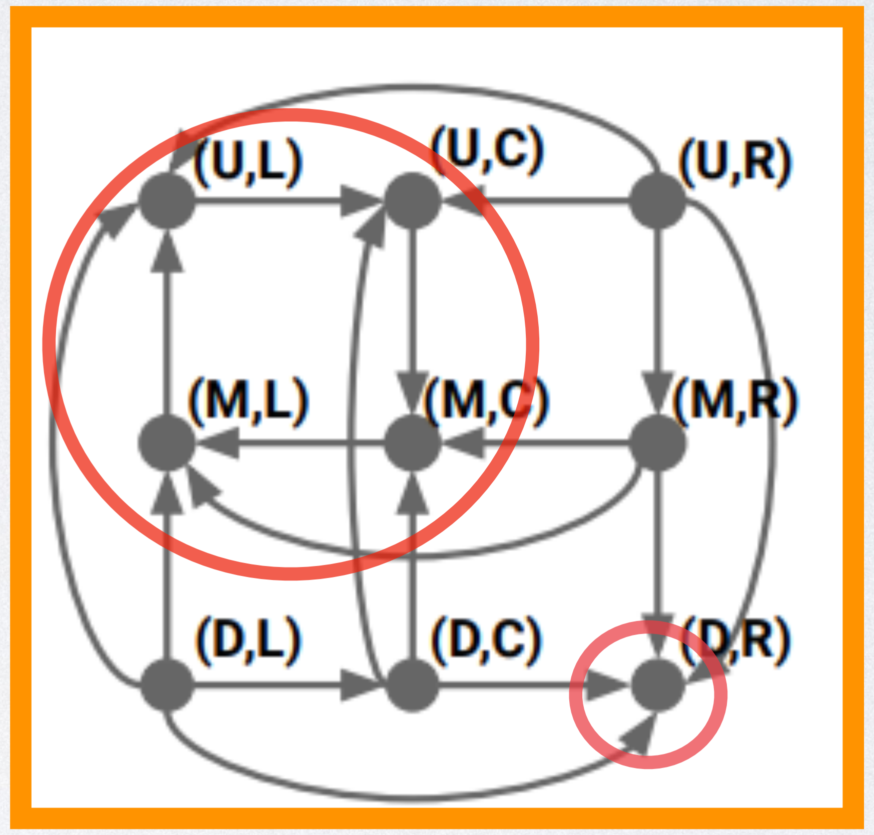
- The response graph of a game is the graph in which the nodes are joint strategy profiles, edges indicates if the deviating player can achieve larger reward.

- Response graph assume one player changes its policy at each time. The graph is sparse!

Game

		II		
		L	C	R
	U	2, 1	1, 2	0, 0
I	M	1, 2	2, 1	1, 0
	D	0, 0	0, 1	2, 2

Response Graph

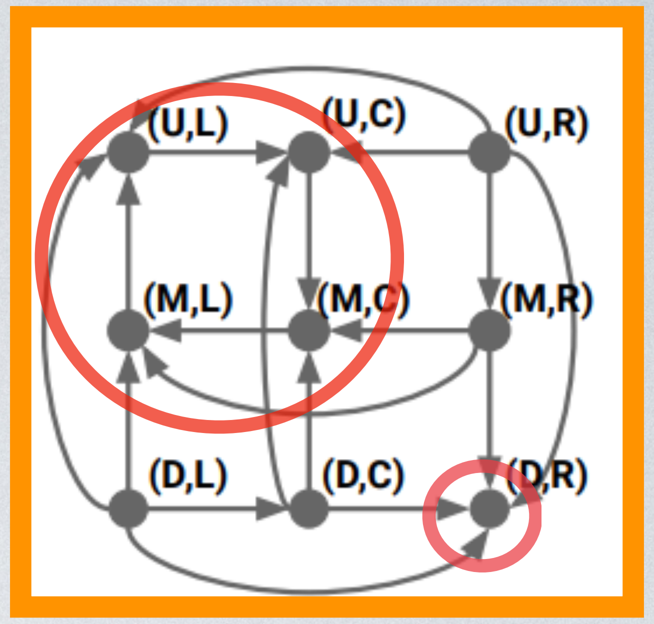


two SSCC here.

- The Sink Strongly Connected Component (SSCC) of the response graph is the subset of nodes in which there are no outbound edges but only inbound edges.

- A node in the flow is either a part of a "*recurrent chain*", or on its way to a "*recurrent chain*".

Modelling the SSCC through a Markov Chain



- SSCC captures the long-term dynamical interactions between agents.
- On the response graph, considering a random walk, following the edges, no matter which node you start from, you will end up converging to the SSCC.
- This process can be modelled through a Markov Chain, and the stationary distribution of the Markov Chain is exactly SSCC.
- To make sure the stationary distribution exists and unique. The chain has to be *irreducible*, meaning every nodes can "travel" to every other nodes.
- To meet such requirement, α-Rank creates a so-called, *Markov-Conley chain*, where the edges are "soft".

α-Rank Algorithm

- α-Rank [Shayegan et al 2019] defines the transitional probability between nodes by

$$\rho_{\pi_{i,a},\hat{\pi}_{i,b}}\left(\pi_{-i}\right) = \frac{1 - e^{-\alpha\left(\mathscr{P}_i\left(\pi_{i,a},\pi_{-i}\right) - \mathscr{P}_i\left(\hat{\pi}_{i,b},\pi_{-i}\right)\right)}}{1 - e^{-m\alpha\left(\mathscr{P}_i\left(\pi_{i,a},\pi_{-i}\right) - \mathscr{P}_i\left(\hat{\pi}_{i,b},\pi_{-i}\right)\right)}}$$

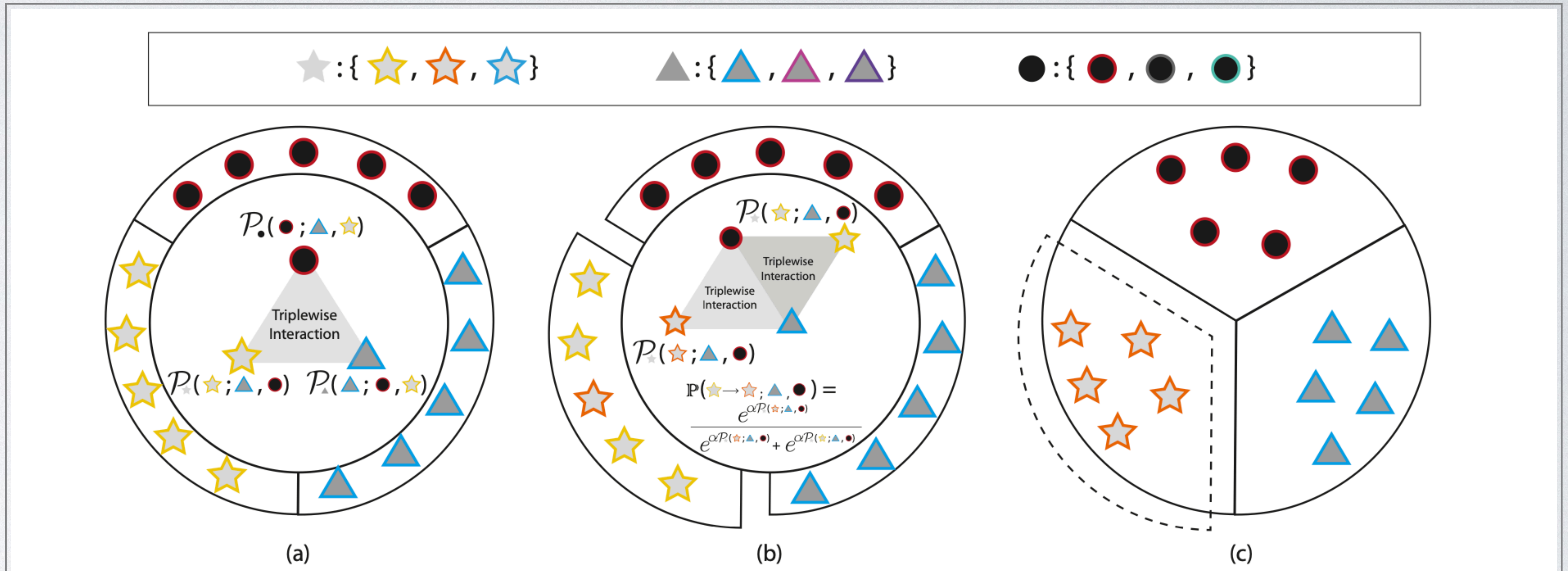


Figure 1: Example of population based evaluation on $N = 3$ players (star, triangle, circle) each with $|s| = 3$ strategies (denoted by the colours) and $m = 5$ copies. a) Each population obtains a fitness value $\mathcal{P}_i$ depending on the strategies chosen, b) one mutation strategy (red star) occurs, and c) the population either selects the original strategy, or being fixated by the mutation strategy.

- Physical meaning of $\rho_{\pi_{i,a},\hat{\pi}_{i,b}}\left(\pi_{-i}\right)$ can be thought of as an evolutionary process above.

transition probability of the Markov Chain

$$[T]_{\pi_{\text{join}},\hat{\pi}_{\text{joint}}} = \begin{cases} \frac{1}{\sum_{l=1}^{N}(k_l-1)}\rho_{\pi_{i,a},\hat{\pi}_{i,b}}\left(\pi_{-i}\right), & \text{if } \left|\pi_{\text{joint}} \backslash \hat{\pi}_{\text{joint}}\right| = 1 \\ 1 - \sum_{\hat{\pi}\neq\pi_{\text{jont}}}[T]_{\pi_{\text{joint}},\hat{\pi}}, & \text{if } \pi_{\text{joint}} = \hat{\pi}_{\text{joint}} \\ 0, & \text{if } \left|\pi_{\text{joint}} \backslash \hat{\pi}_{\text{joint}}\right| \geq 2 \end{cases}$$

α-Rank Algorithm

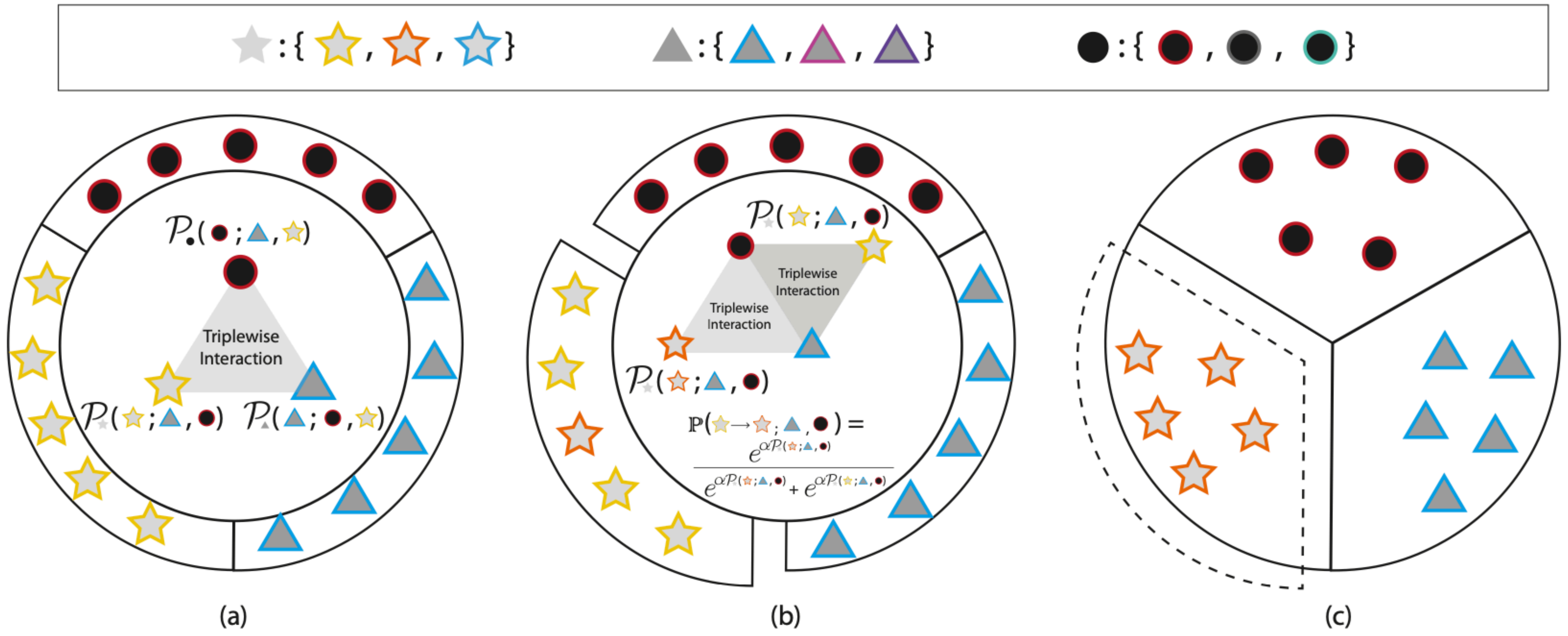
- α-Rank uses α in $\rho_{\pi_{i,a},\hat{\pi}_{i,b}}\left(\pi_{-i}\right)$ to control the "softness" of edges in the response graph, so that the Markov Chain can be irreducible.
- α means how likely a sub-optimal joint strategy is going to dominate an optimal joint strategy. In experiments, it is usually set as a large number.
- The unique stationary distribution of the Markov chain is

$$\boldsymbol{v} = \lim_{t\to\infty} [T]^t \boldsymbol{v}_0$$

- The rank of probability mass of $\boldsymbol{v}$ is the output of α-Rank. Computing $\boldsymbol{v}$ is polynomial-time.
- The physical meaning is the evolutionary strength/stability of joint strategy profile in terms of how strong it can resisting mutations's invasions. Caveat: this is not the same idea as ESS.
- The connection of α-Rank equilibrium to Nash equilibrium/ESS is unclear yet.

α-Rank Summary

- α-Rank answers the question of how to evaluate/rank joint-policies.
  - A solution concept based on Conley's theorem & graph theory.
    - it can model recurrent chains (limited cycles) in dynamical system, e.g. Rock-Paper-Scissor game.
    - it is tractable in multi-player general-sum games.



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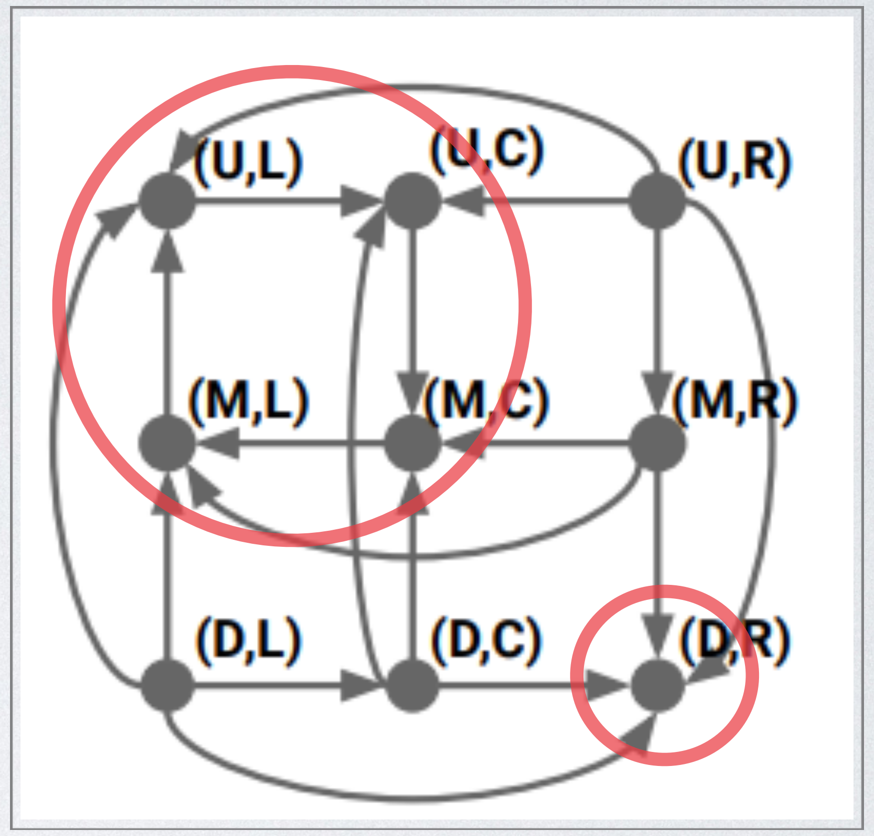
$$\rho_{\pi_{i,a},\hat{\pi}_{i,b}}\left(\pi_{-i}\right) = \frac{1 - e^{-\alpha\left(\mathscr{P}_i\left(\pi_{i,a},\pi_{-i}\right) - \mathscr{P}_i\left(\hat{\pi}_{i,b},\pi_{-i}\right)\right)}}{1 - e^{-m\alpha\left(\mathscr{P}_i\left(\pi_{i,a},\pi_{-i}\right) - \mathscr{P}_i\left(\hat{\pi}_{i,b},\pi_{-i}\right)\right)}}$$

Example:

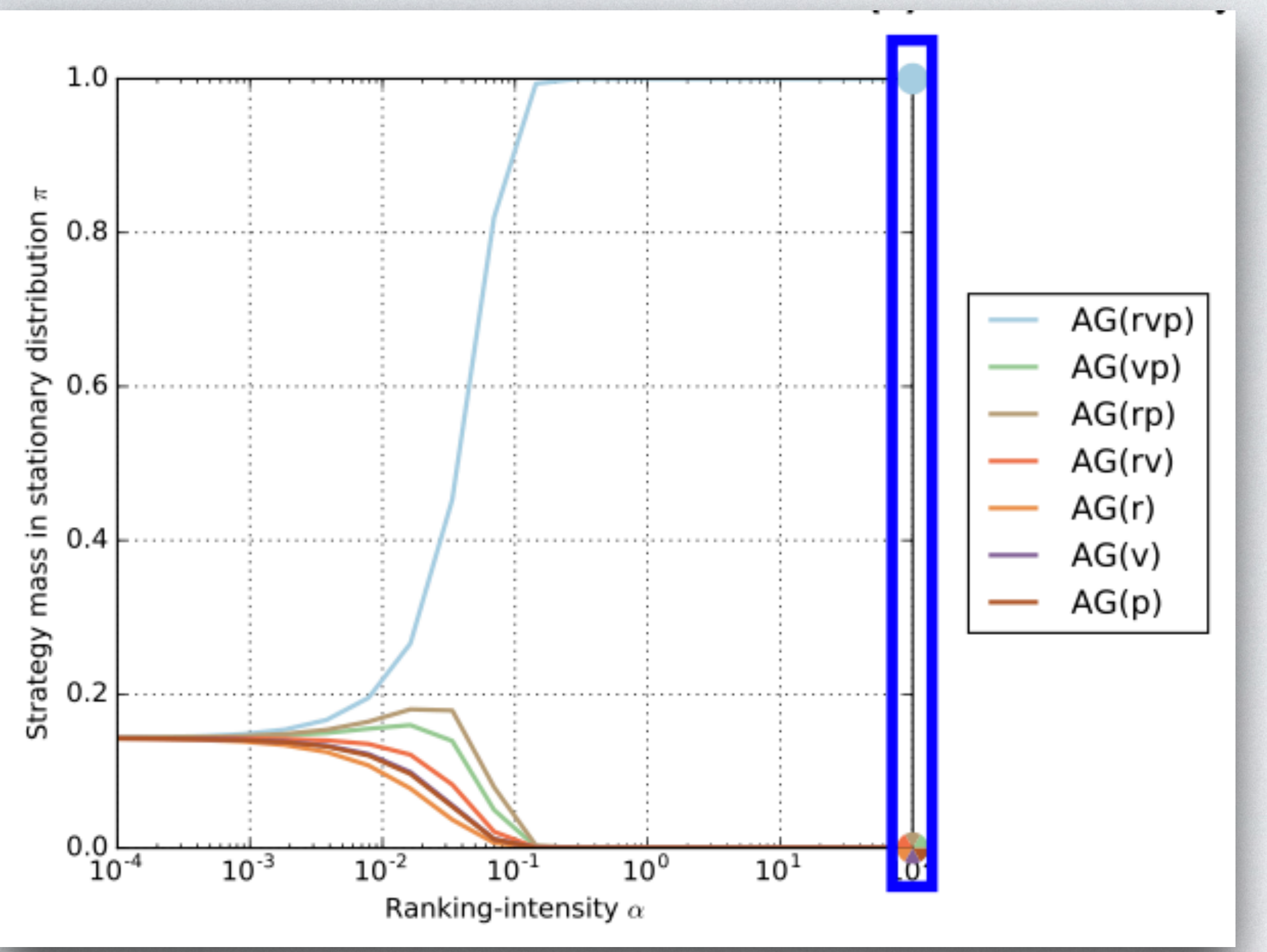
		II: L	II: C	II: R
I	U	2, 1	1, 2	0, 0
I	M	1, 2	2, 1	1, 0
I	D	0, 0	0, 1	2, 2

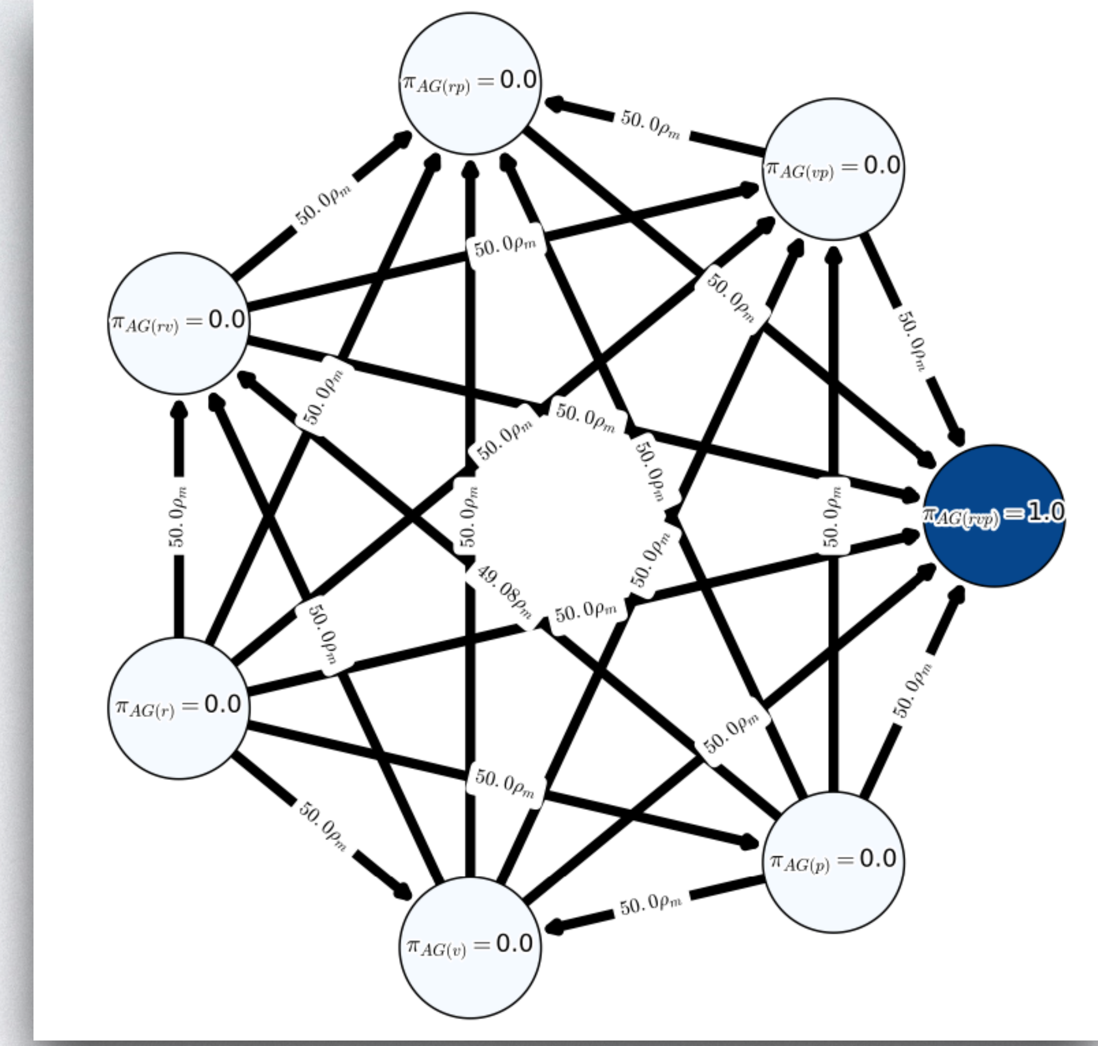


1. Collect the pay-off values for different strategy profiles.
2. Construct the Markov Chain based on $\rho_{\pi_{i,a},\hat{\pi}_{i,b}}\left(\pi_{-i}\right)$
3. Compute the stationary distribution $\boldsymbol{\nu} = \lim_{t\to\infty}[T]^t \boldsymbol{\nu}_0$
4. Rank the joint strategy profile based on probability of $\boldsymbol{\nu}$.

α-Rank Results

AlphaGo version comparison



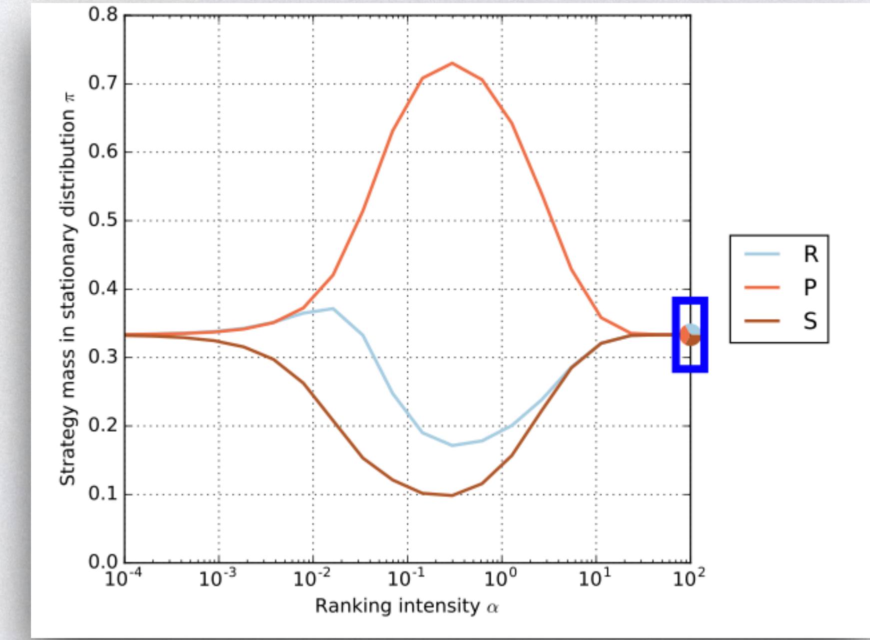


Agent	Rank	Score
AG(rvp)	1	1.0
AG(vp)	2	0.0
AG(rp)	2	0.0
AG(rv)	2	0.0
AG(r)	2	0.0
AG(v)	2	0.0
AG(p)	2	0.0

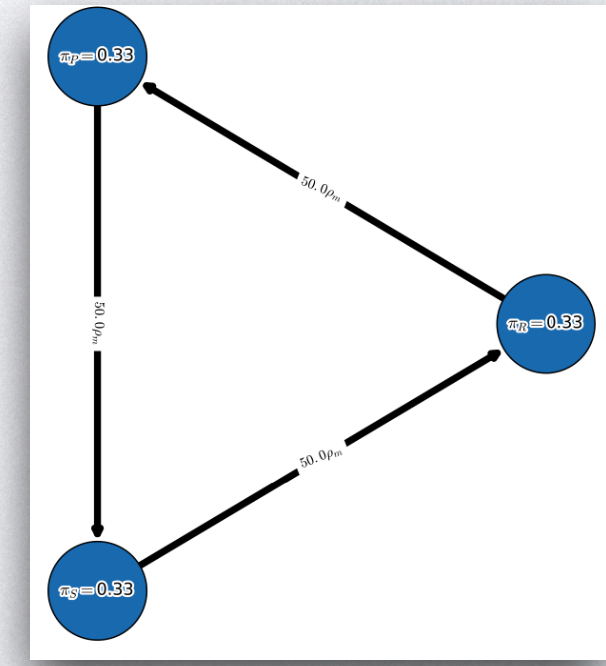
Biased RPS

	R	*P*	*S*
R	0	−0.5	1
P	0.5	0	−0.1
S	−1	0.1	0

(a) Payoff matrix.



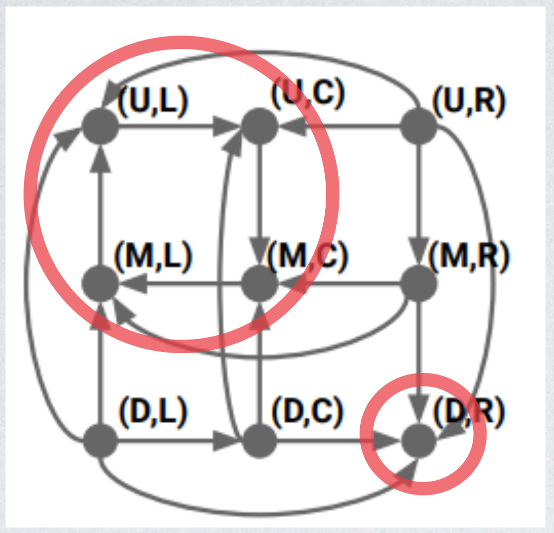
Agent	Rank	Score
R	1	0.33
P	1	0.33
S	1	0.33



α^{α}-Rank: A Scalable Solution for α-Rank [Yang 2020]

Example:

		II		
		L	C	R
I	U	2, 1	1, 2	0, 0
	M	1, 2	2, 1	1, 0
	D	0, 0	0, 1	2, 2



1. Collect the pay-off values for different strategy profiles.

2. Construct the Markov Chain based on $\rho_{\pi_{i,a},\hat{\pi}_{i,b}}(\pi_{-i})$

3. Compute the stationary distribution $\boldsymbol{\nu} = \lim_{t\to\infty} [T^T]^t \boldsymbol{\nu}_0$

4. Rank the joint strategy profile based on probability of $\boldsymbol{\nu}$.

Conclusion:

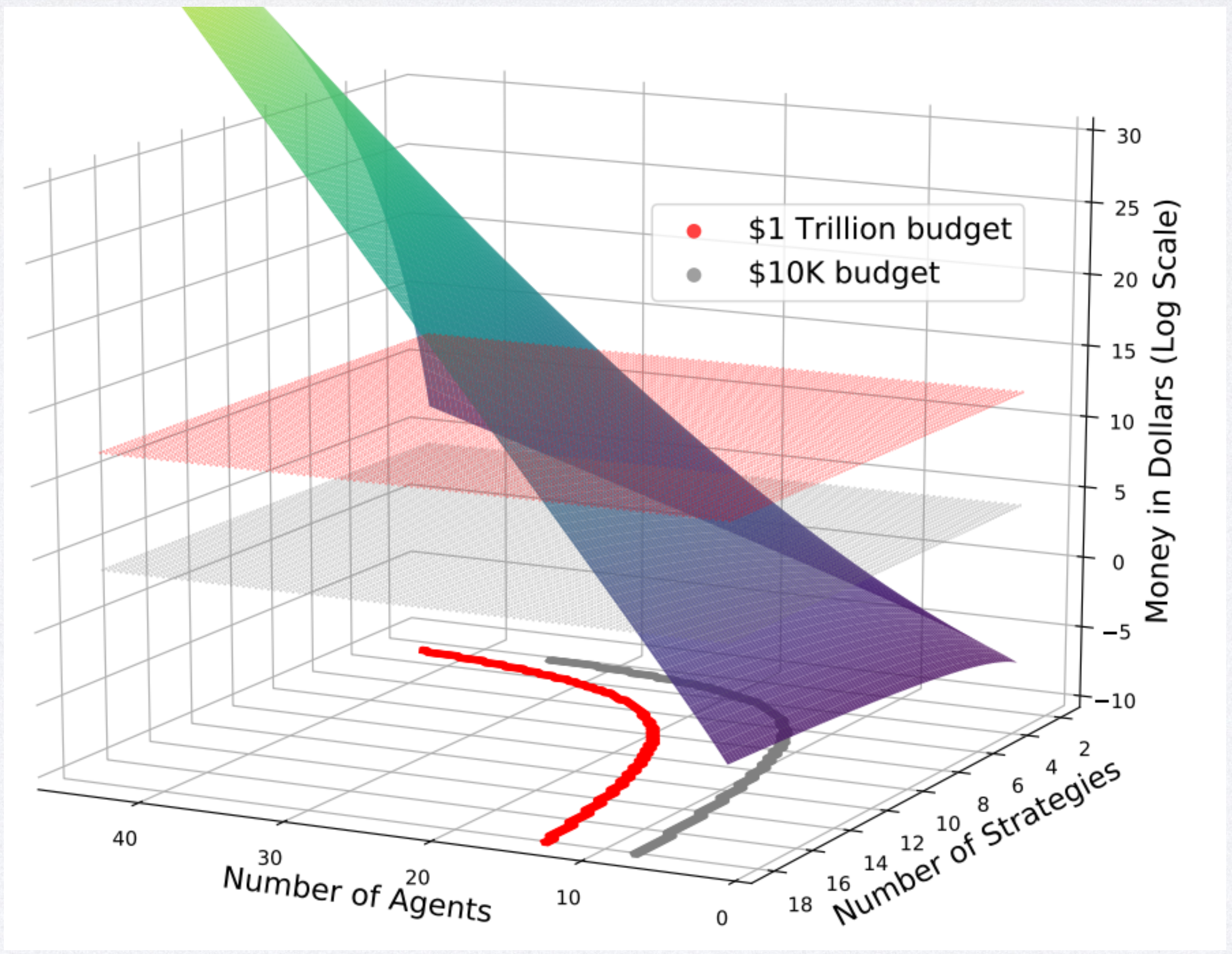
1. We conjecture that solving α-Rank is still **NP-Hard** because the size of the Markov Chain is exponential to the number of agents.

2. A polynomial-time solver on exponential-sized input cannot be claimed as tractable.

3. Take TSP as example, one cannot claim a NP-Hard problem solvable by just creating an exponentially-sized input.

Game Env.	PetaFlop/s-days	Cost ($)	Time (days)
AlphaZero Go [29]	$1,413 \times 7$	$207M$	$1.9M$
AlphaGo Zero [28]	$1,181 \times 7$	$172M$	$1.6M$
AlphaZero Chess [29]	17×1	$352K$	$3.2K$
MuJoCo Soccer [18]	0.053×10	$4.1K$	72
Leduc Poker [15]	0.006×9	420	7
Kuhn Poker [11]	$< 10^{-4} \times 256$	< 1	–
AlphaStar [31]	52, 425	$244M$	$1.3M$

Cost of Step 1



Cost of Step 2

Table 1: Time and space complexity comparison given N(number of agents) $\times$ k(number of strategies) table as inputs.

Method	Time	Memory
Power Method	$O(k^{N+1}N)$	$O(k^{N+1}N)$
PageRank	$O(k^{N+1}N)$	$O(k^{N+1}N)$
Eig. Decomp.	$O(k^{N\omega})$	$O(k^{N+1}N)$
Mirror Descent	$O(k^{N+1}\log k)$	$O(k^{N+1}N)$

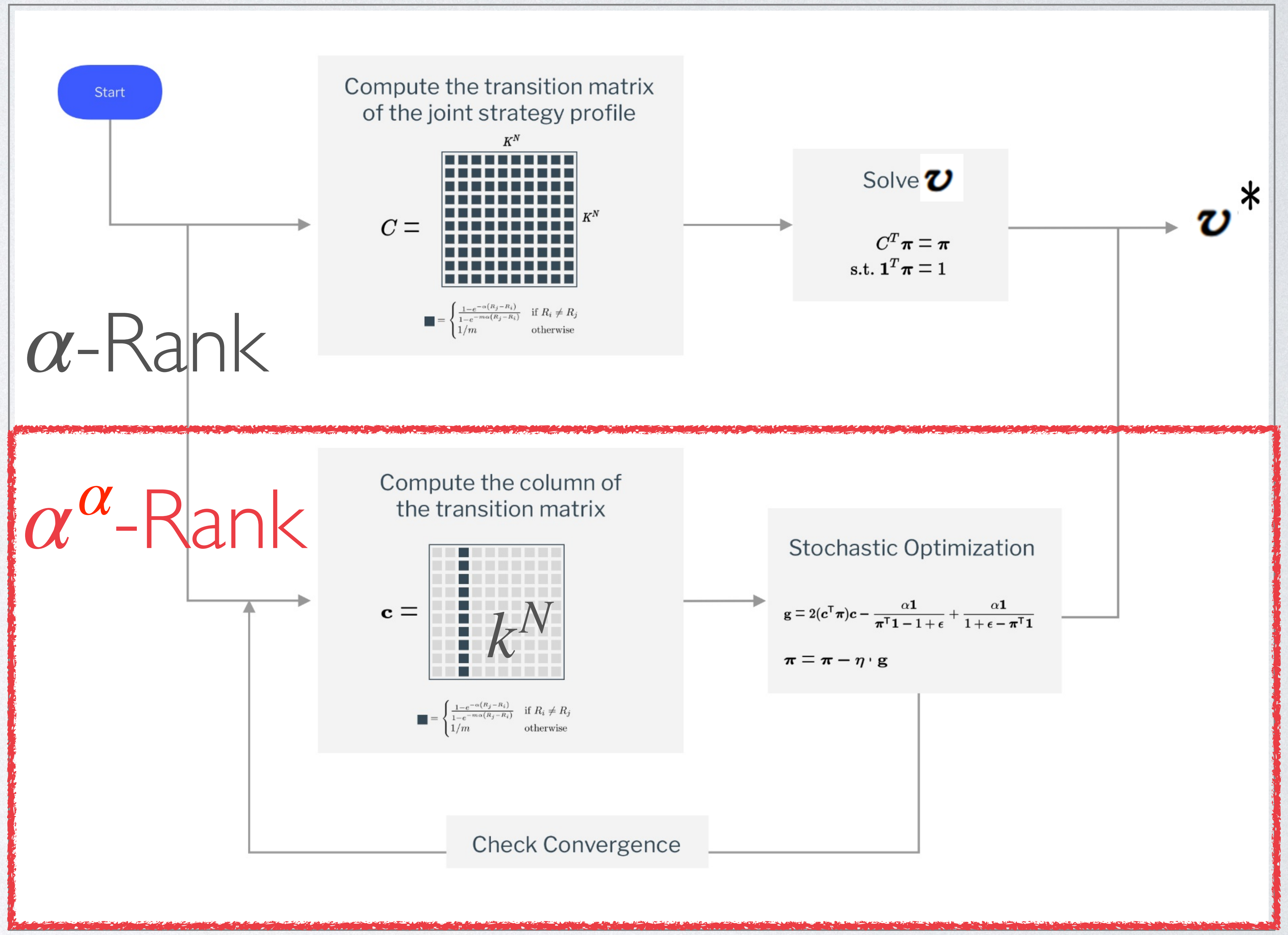
Cost of Step 3

α^{α}-Rank: A Scalable Solution for α-Rank [Yang 2020]

- Novelty 1: reformulate as a stochastic optimisation problem
  - Though cannot improve the time-complexity, but now can do early stopping for large meta-game solutions.
  - Saves time in getting the payoff values for the transition matrix of Markov chain.

Table 1: Time and space complexity comparison given N(number of agents) $\times$ k(number of strategies) table as inputs.

Method	Time	Memory
Power Method	$\mathcal{O}\left(k^{N+1}N\right)$	$\mathcal{O}\left(k^{N+1}N\right)$
PageRank	$\mathcal{O}\left(k^{N+1}N\right)$	$\mathcal{O}\left(k^{N+1}N\right)$
Eig. Decomp.	$\mathcal{O}\left(k^{N\omega}\right)$	$\mathcal{O}\left(k^{N+1}N\right)$
Mirror Descent	$\mathcal{O}\left(k^{N+1}\log k\right)$	$\mathcal{O}\left(k^{N+1}N\right)$



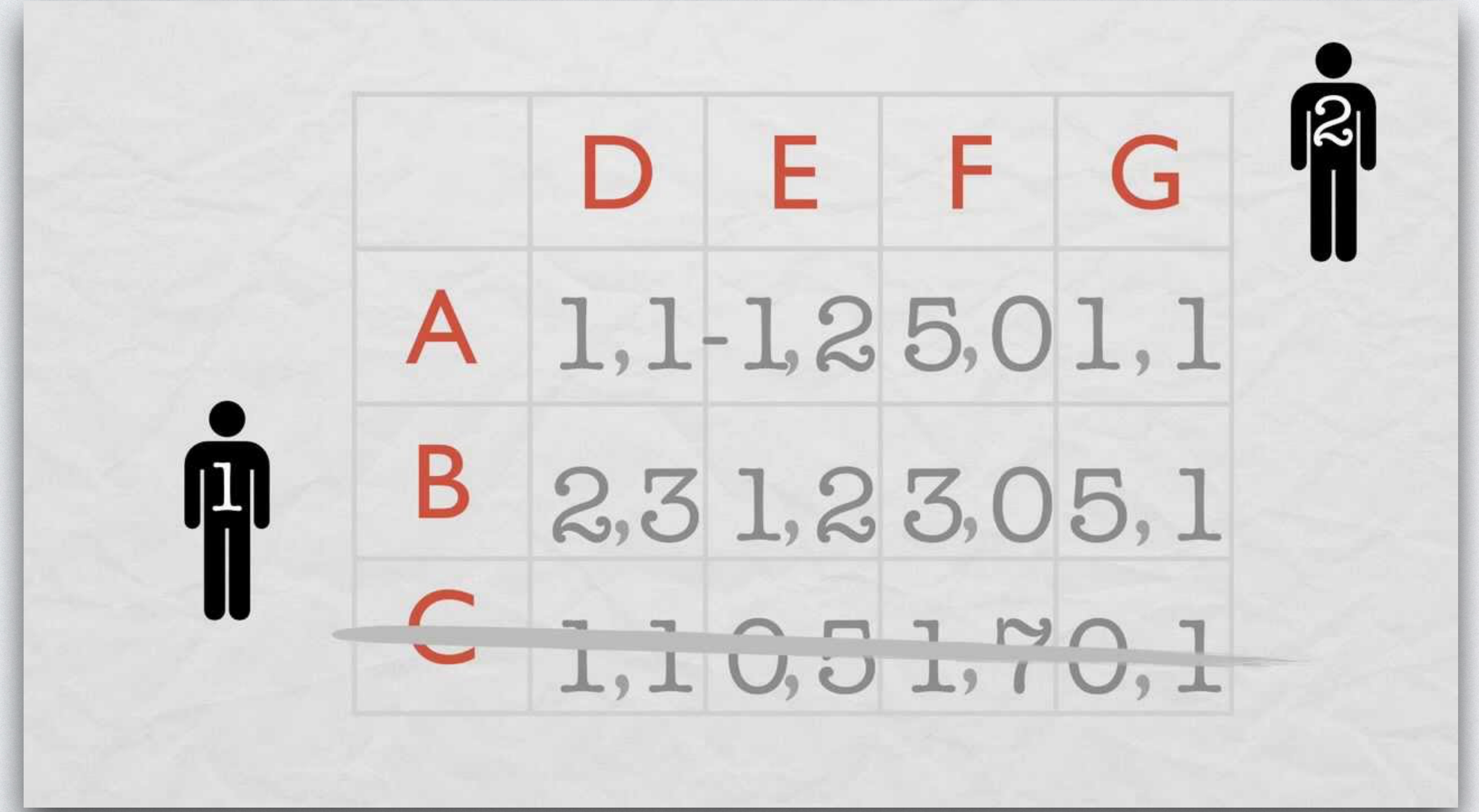
$$v = \lim_{t\to\infty} [T]^t v_0$$

$$\min_{v\in\mathbb{R}^n} \frac{1}{n}\sum_{i=1}^{n}\left(v^T c_i\right)^2 - \lambda \log\left(\delta^2 - \left[v^T\mathbf{1} - 1\right]^2\right) - \frac{\lambda}{n}\sum_{i=1}^{n}\log\left(v_i\right)$$

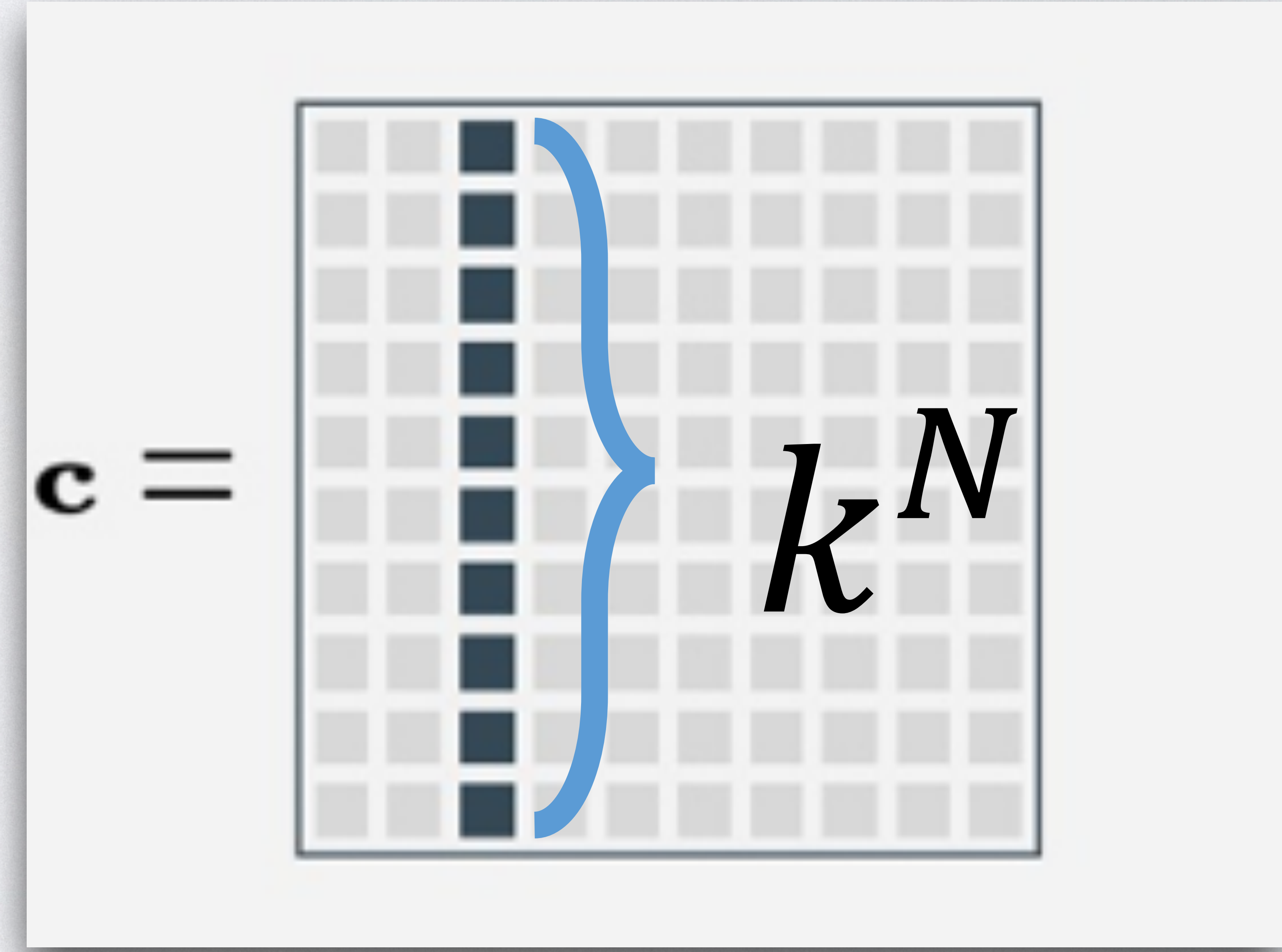
Adam/SGD/…

α^{α}-Rank: A Scalable Solution for α-Rank [Yang 2020]

- Novelty 2: Introducing a heuristics to start with a subset of strategies and then increasingly expand the strategy space of each agent, we can decrease k further.
  - Intuition: remove dominated strategy from the beginning and save the exploration time, and add any good strategy back if we miss them wrongly in the initialisation.



All joint strategy profile involving “C" will not be SSCC, removing “C” can save exploration time.



Still exponential size, make k smaller

Scalability of α^α-Rank on Large Meta-games

Random matrices

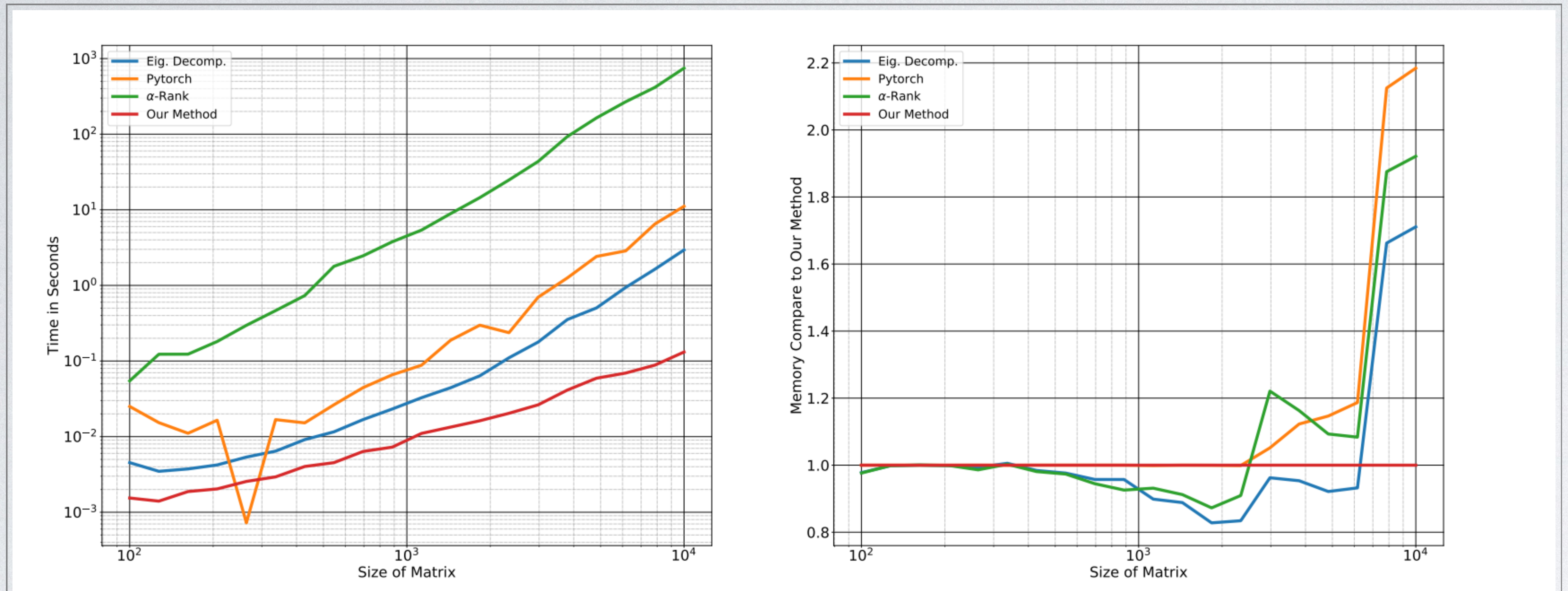
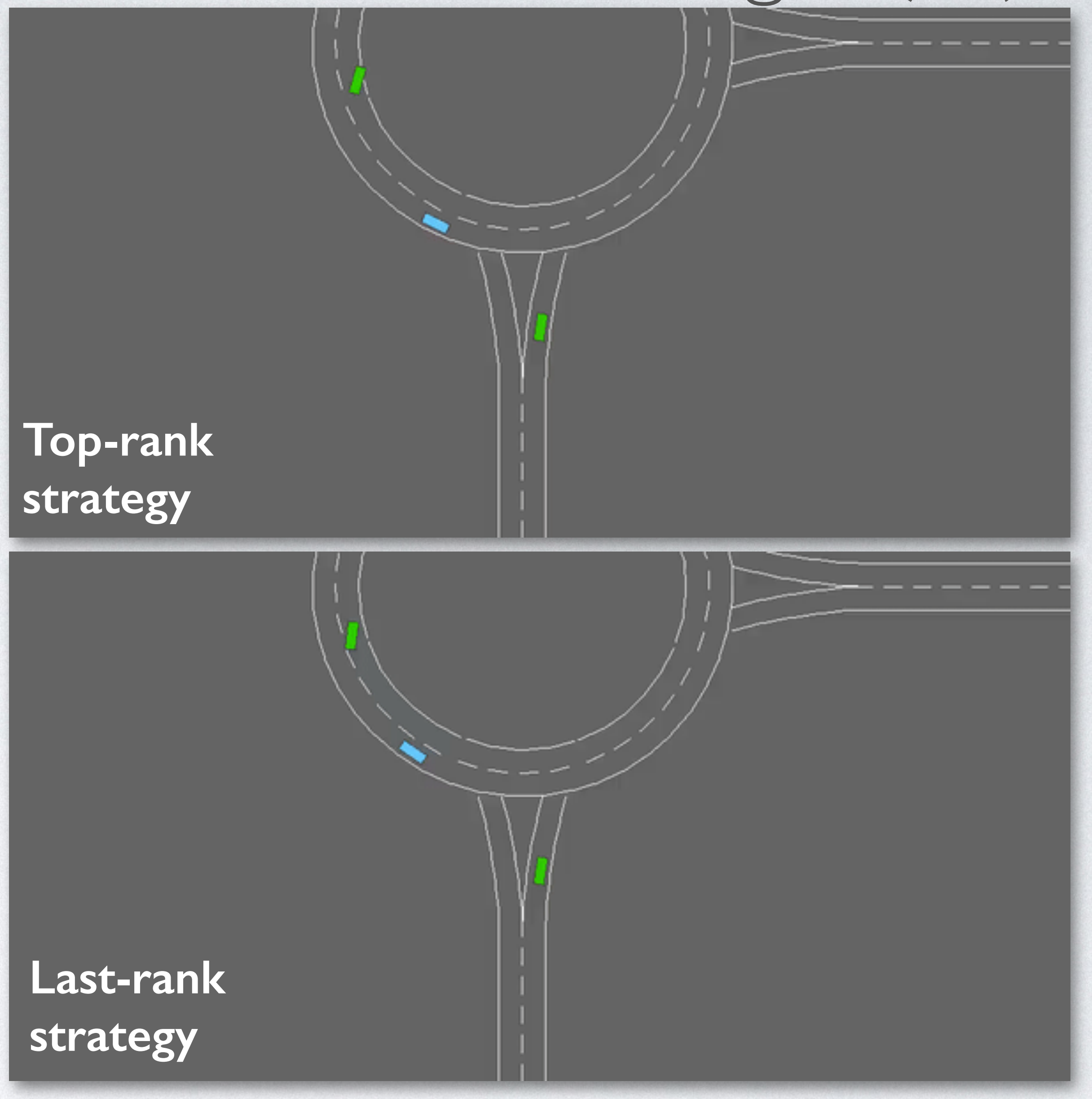
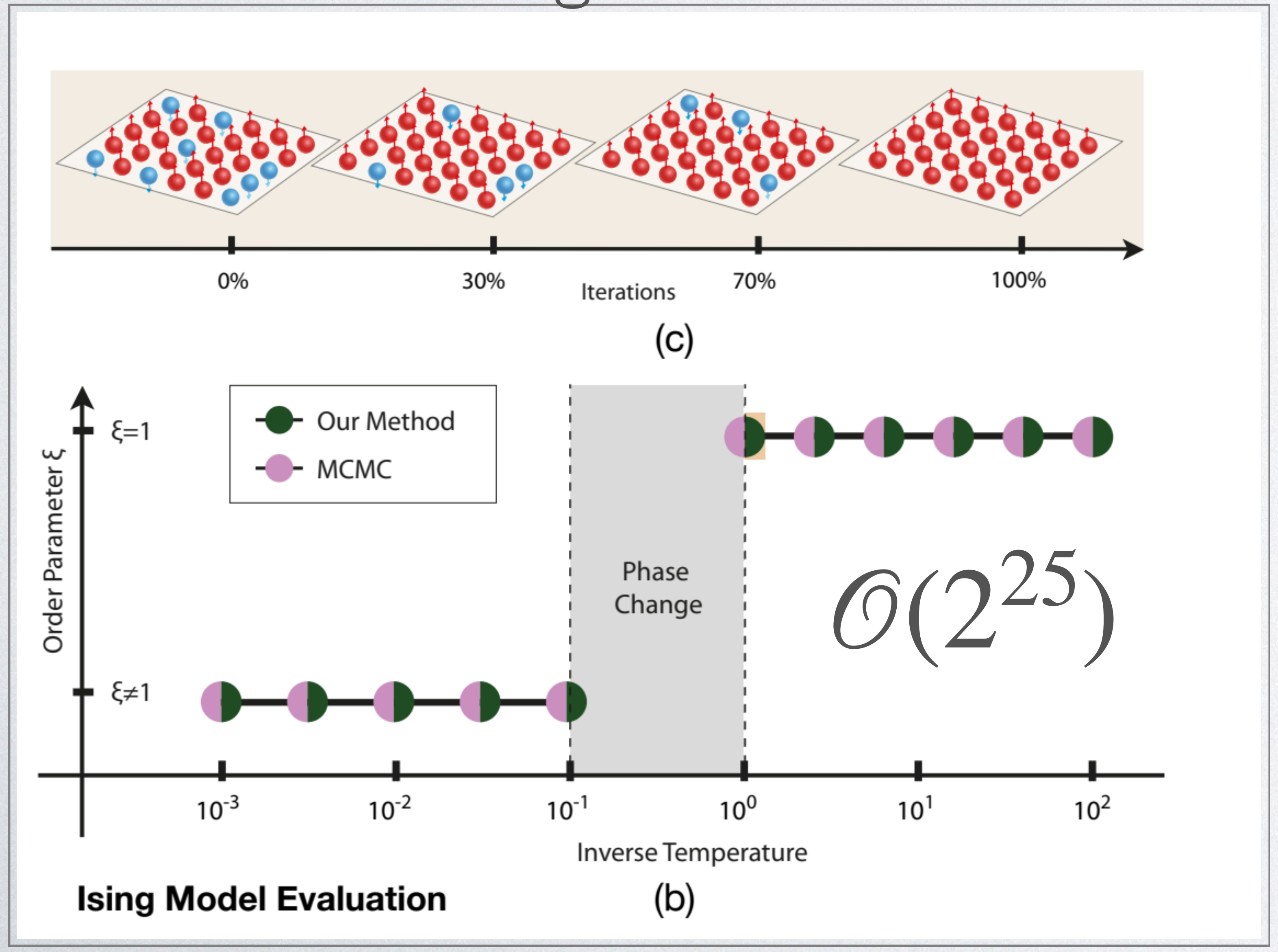


Figure 4: Comparisons of time and memory complexities on varying sizes of random matrices.

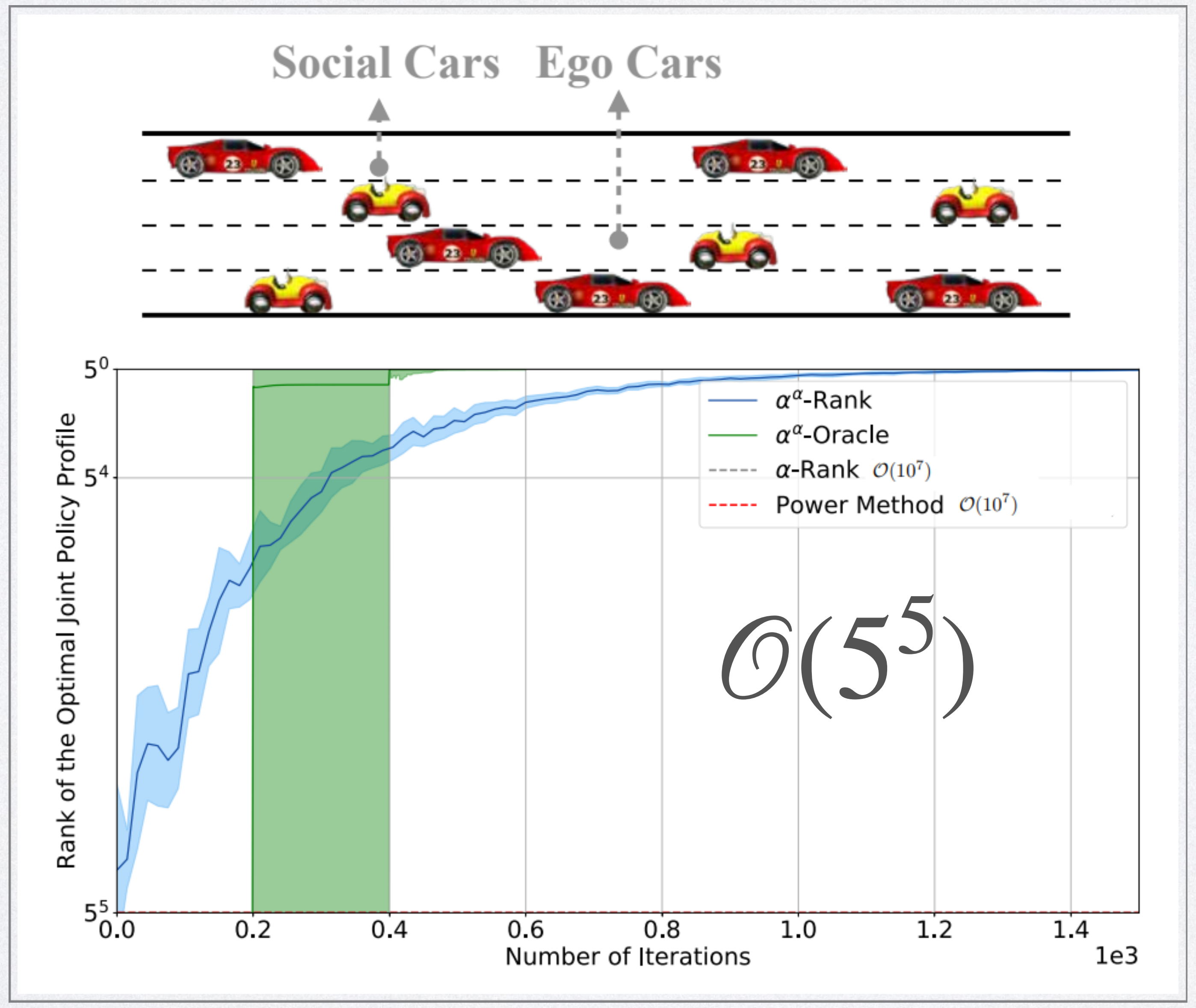
Roundabout driving $\mathcal{O}(5^3)$



Ising Model



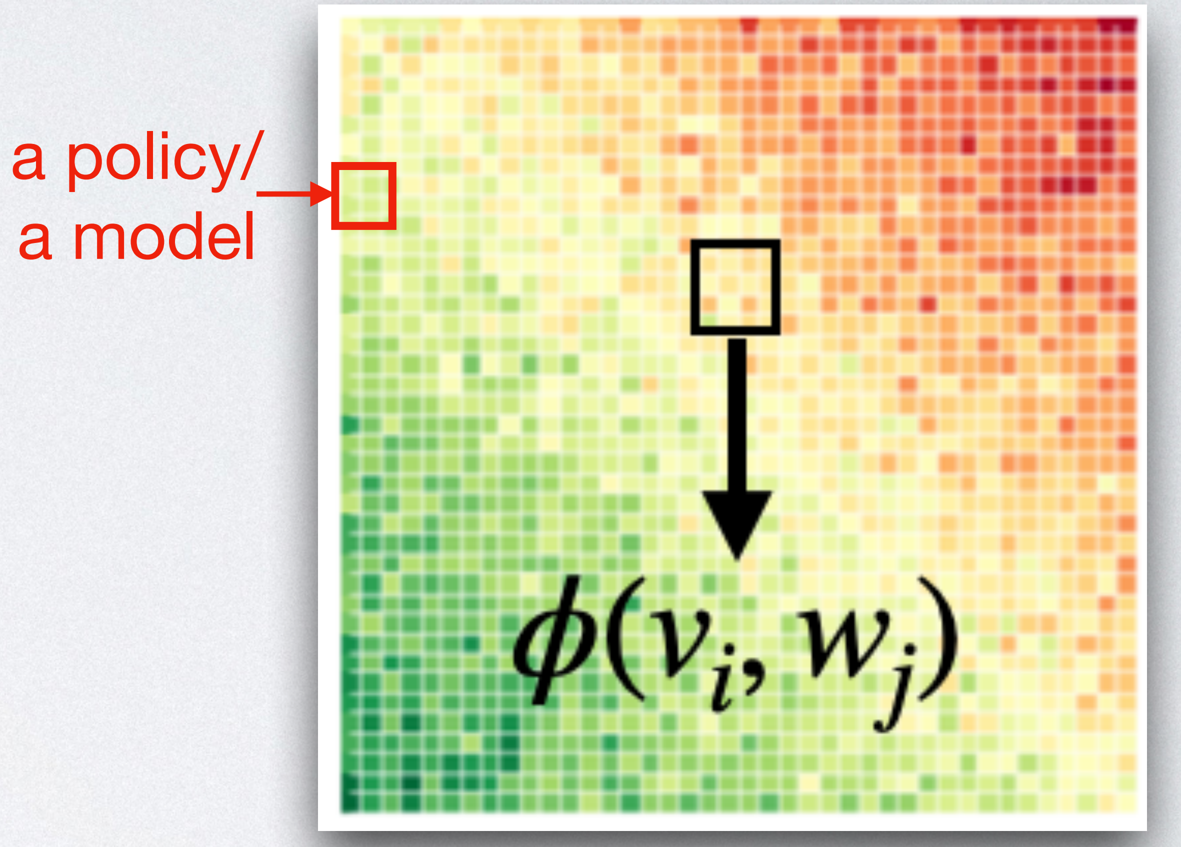
Autonomous driving



Highway Driving $\mathcal{O}(10^5)$

Summary of Meta-game Policy Evaluation

- Give a meta-game with fixed set of players and strategies, we have introduced methods to answer the questions of which joint strategy profile is "optimal", specifically, we can know
  - what is definition of "optimality"
  - which metric suits transitive/in-transitive games
  - which metric is tractable in multi-player games
  - which metric can deal with general-sum games
  - which metric can induce stable equilibrium
  - which metric can induce unique equilibrium
  - which metric can model the flow of dynamics or being a fixed point



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THANKS!